Measuring dynamic efficiency under uncertainty:

A simulation study

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Abstract

Since 2003, measuring efficiency in dynamic contexts has received considerable attention. Dynamic efficiency analysis accounts for both the interdependency of production decisions over time, as well as adjustment costs, and also distinguishes between variable and quasi-fixed inputs in the production process. However, structural models of dynamic efficiency have thus far ignored uncertainty; this may lead to misleading measures of efficiency. Uncertainty affects the optimal allocation of input decisions and it is particularly true for the optimal adjustment of quasi-fixed factors over time. Hence, to fill this gap, this thesis aims to develop a theoretical model of dynamic efficiency under uncertainty based on the cost-minimization problem.

To derive such a model, the author uses two components, namely the static shadow cost approach and a stochastic dual model of investments under uncertainty. The shadow cost approach allows one to disentangle economic inefficiency into technical and allocative inefficiency, while the stochastic intertemporal duality model enables one to consider uncertainty and adjustment costs. Formulating an empirical model requires one to specify the functional form of the respective value function. Here, the specified value function properties facilitate output and price uncertainty to influence optimal factor demand equations. The resulting empirical stochastic factor demand equations then serve as a starting point for the econometric estimation of technical and allocative inefficiency measures.

Theoretical findings from the derived model were subsequently tested using a simulation, to determine how large the omitted variable bias is on the estimates of the coefficients if uncertainty is ignored in optimal factor allocations, and to analyze the influence of uncertainty on factor demand equations. The simulation results reveal that disregarding uncertainty in optimal factor allocations leads to biased estimates of model parameters. Quasi-fixed factor price uncertainty has a negative impact on the investment demand equation for the different values of the quasi-fixed factor level, whereas variable input price uncertainty has a negative marginal effect on the variable input demand equation for various combinations of investment and quasi-fixed factor levels.

Keywords: dynamic efficiency, uncertainty, technical and allocative efficiency, simulation

Zusammenfassung

Seit 2003 hat die Effizienzmessung im dynamischen Kontext erheblich an Aufmerksamkeit gewonnen. Die dynamische Effizienzanalyse berücksichtigt sowohl die zeitliche Interdependenz der Produktionsentscheidungen als auch Anpassungskosten. Zudem wird zwischen variablen und quasi-fixen Produktionsfaktoren unterschieden. Allerdings haben strukturelle dynamische Effizienzmodelle bisher Unsicherheit vernachlässigt, was zu irreführenden Effizienzwerten führen kann. Unsicherheit beeinflusst die optimale Anpassung von Produktionsentscheidungen; dies ist besonders relevant für die optimale Anpassung der quasi-fixen Faktoren im Zeitablauf. Deshalb ist es das Ziel dieser Doktorarbeit, diese Lücke zu schließen und ein theoretisches Modell für die dynamische Effizienzmessung unter Unsicherheit basierend auf einer Kostenminimierung zu entwickeln.

Um ein solches Modell herzuleiten, verwendet die Autorin zwei Komponenten: den statischen Schattenkostenansatz und ein stochastisches duales Investitionsmodel unter Unsicherheit. Während der Schattenkostenansatz die ökonomische Effizienz in eine technische und eine allokative Komponente zerlegt, erlaubt das stochastische intertemporale Dualitätsmodell, Unsicherheit und Anpassungskosten zu berücksichtigen. Hierbei ermöglichen die Eigenschaften der Zielfunktion, dass Output- und Preisunsicherheit die optimalen Faktornachfragegleichungen beeinflussen. Die resultierenden empirischen stochastischen Nachfragegleichungen dienen als Grundlage für die ökonometrische Schätzung der technischen und allokativen Effizienz.

Die theoretischen Erkenntnisse des hergeleiteten Modells wurden anschließend mit Hilfe einer Simulation überprüft, mit dem Ziel, einerseits die Höhe der Verzerrung der geschätzten Koeffizienten durch ausgelassene Variablen zu ermitteln, wenn Unsicherheit bei der optimalen Faktoranpassung vernachlässigt wird, und andererseits den Einfluss der Unsicherheit auf die Faktornachfragegleichungen zu analysieren. Die Simulationsergebnisse zeigen, dass eine Vernachlässigung der Unsicherheit zur Verzerrung der geschätzten Modellparameter führt. Die Preisunsicherheit der quasi-fixen Produktionsfaktoren beeinflusst die Investitionsnachfrage negativ für unterschiedlich hohe Ausprägungen des quasi-fixen Kapitalstocks. Die Preisunsicherheit der variablen Produktionsfaktoren hat einen negativen Einfluss auf die variablen Faktornachfragen für verschiedene Kombinationen von Investitionen und quasi-fixen Faktoren.

Schlagwörter: dynamische Effizienz, Unsicherheit, technische und allokative Effizienz, Simulation

Table of contents

List of abbreviations

Acronyms

AE	Allocative Efficiency
AI	Allocative Inefficiency
DEA	Data Envelopment Analysis
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
HJB	Hamilton-Jacobi-Bellman
ML	Maximum Likelihood
OLS	Ordinary Least Square
RESET	Regression Specification Error Test
TE	Technical Efficiency
TI	Technical Inefficiency
U.S.	United States
et al.	*et alii*
i.e.	that is
cf.	confer

List of symbols

α	vector of drift parameters
β	parameter(s) of the variable input demand
γ	short-run marginal cost
δ	depreciation rate
ε	error term
η	parameter(s) of the numeraire input demand
θ	parameter(s) of investment demand with uncertainty
ι	output-oriented TE measure
κ	parameter(s) of investment demand without uncertainty
λ_{21}	AI of the variable input price
μ	AI of net investment
ν	parameter(s) in the auxiliary regression of investment demand
ξ_1	GARCH parameter of the conditional variance equation
ρ	slope parameter of the production function
σ	standard deviation

σ^2	variance
τ	input-oriented TE measure
τ_K	TI of net investment
τ_x	TI of the variable input quantity
φ	Lagrangian multiplier
Ψ	vector of variance parameters
ω_0	constant parameter of the conditional variance equation
ω_1	ARCH parameter of the conditional variance equation
Δ	difference operator
$\sqrt{\Delta t}$	length of time interval
Σ	matrix of variance and co-variance parameters
Ω	uncertainty
A	second order parameters of the behavioral value function
B	constant parameter of the production function
C	variable cost function
$D_I(y,x)$	input distance function
$D_O(x,y)$	output distance function
E_0	expectation operator
$F(\cdot)$	production function
GR	production technology
I	gross investment
$J(\cdot)$	value function
K	quasi-fixed input level
K_{it-1}	lag of quasi-fixed input level
$\dot{K}$	net investment
$\dot{K}^*$	optimal net investment demand
L	Lagrangian problem
$L(y)$	input requirement set
M	second order parameter of the behavioral value function
N	adjustment rate
a_0	constant parameter of the behavioral value function
a_t	shock or innovation of an asset return at time *t*
b	first order parameters of the behavioral value function
c	quasi-fixed input price

dv	Wiener increment
e_t	error term
$h\left(y, w_n^b\right)$	cost minimizing input vector
i	individuals or firms
j	elements of the state vector (z)
ln	natural logarithm
m	number of quasi-fixed inputs
min	minimization operator
n	number of variable inputs
r	discount rate
t	time
w	variable input price
w^b	shadow price of variable input
x	variable input level
x^*	optimal variable input demand
y	output
z	state vector
$\forall$	for all
$\sum$	summation operator
∂	partial derivative
∞	infinity
SE	standard error
superscript 'a'	actual
superscript 'b'	behavioral
superscript 'o'	optimized actual
subscripts of F, J and Ω	denote partial derivatives

List of figures

List of tables

1 Introduction

1.1 Motivation

The static modeling of efficiency has experienced substantial theoretical and methodological progress over the last three decades, but little progress has been made in the dynamic context. However, static efficiency analyses assume that a firm will instantly adjust to a long term optimal value of the capital stock without considering the costs associated with adjusting it (Fallah-Fini et al. 2013; Gardebroek and Oude Lansink 2008). As a result, intertemporal dependencies of factor allocations are ignored (Silva and Stefanou 2003). Disregarding the existence of quasi-fixed factor stock, as well as constraints associated with the dynamic production decisions, results in biased estimates of static efficiency measures (Chen and van Dalen 2010; Nemoto and Goto 2003; Skevas et al. 2012). In this situation, efficient firms may appear to be inefficient in their production, therefore, this phenomenon is considered as "seemingly inefficient." For instance, it may be optimal for a particular firm to stick to an outdated technology and sacrifice productivity if investment costs are irreversible, and future returns are random. Similarly, in the case of lacking secondary markets for specific assets, it could be optimal not to reduce the capital stock in response to a decline in marginal capital productivity.

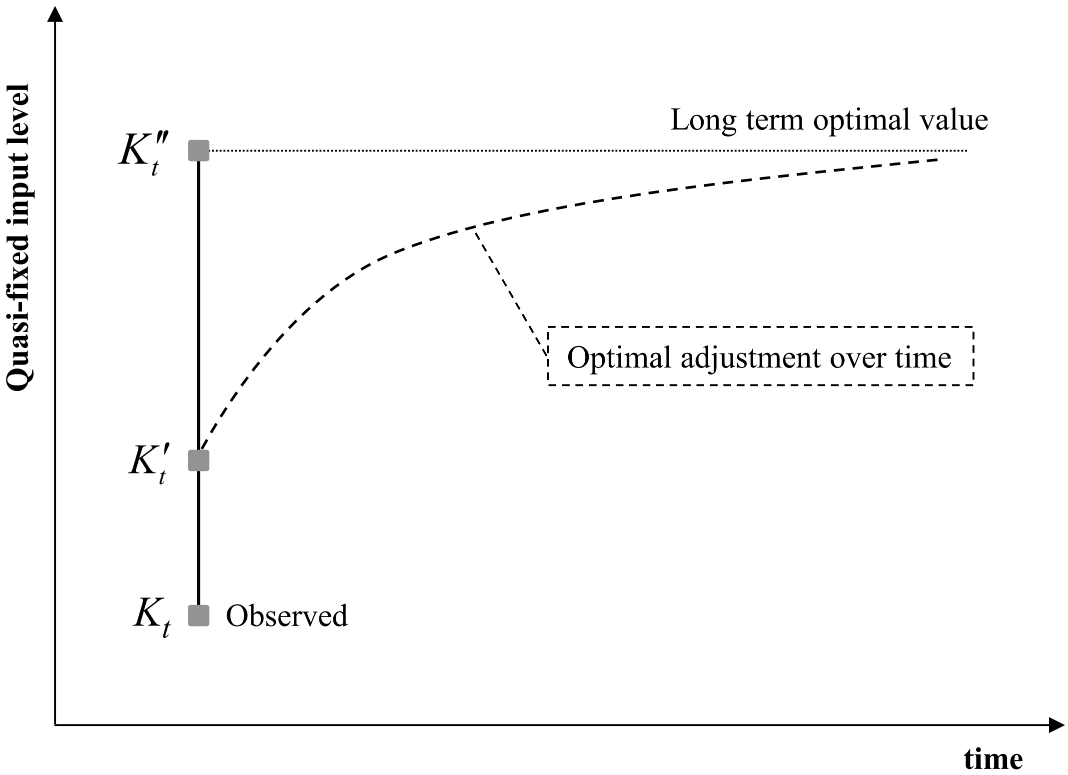
Efficiency needs to be considered in both short-run as well as long-run production decisions; this helps define the appropriate time dimensions of economic decision making (Stefanou 2009). The long run entails a series of short-run situations offered to the firm, in such a way that the firm plans for the long run but always works in the short run. In other words, the long run refers to a firm's planning forward to opt for a future short-run production perspective, whereas in the short run, all economic activities take place but the production factor(s) is considered as fixed—this is because when adjusting the factor level the economic surroundings place a higher cost (Stefanou 2009). Furthermore, the distinction between short run and long run accounts for the decisions on the use of variable inputs that are conditional on the endowment with quasi-fixed assets, and adjustment costs incurred by the firms when changing the quasi-fixed factors. However, the selection of long-run inefficiency cannot be a limiting factor when someone works with a short-run inefficiency measure (Stefanou 2009).

When production decisions are considered in efficiency evaluation over time, the distinction between static and dynamic views of the firm differs. The efficiency definitions are well defined in a static perspective—which generally builds on evaluating the actual performance of the firm with optimal performance embedded in the production (cost, revenue or profit) frontier. In contrast, two definitions of efficiency exist in a dynamic perspective: stock and flow notions of dynamic efficiency measures. Based on these two measures, the efficient allocation of inputs—variable and quasi-fixed factors—takes place in the long-run. First, the stock notion of dynamic efficiency emphasizes capital trajectory as a deviation between observed and optimal trajectories, but this fails to reveal how investment decisions are made (Stefanou 2009). Second, the flow notion of dynamic efficiency is also referred to as temporal efficiency (Stefanou 2009), wherein "the farmer's decisions are assumed to be made in the short-run with a view to the long-run", but this concept fails to assist in clearly distinguishing capital trajectories. The flow notion is conditioned on past investment decisions, but it shows intertemporal linkages of past decisions to future prospects.

Figure 1.1 illustrates the efficiency measure in a static and a dynamic view of the firm (cf. Gardebroek and Oude Lansink 2008), where the variable K_t represents the observed level of quasi-fixed factor on a firm in a specific time t, where K_t' is the optimal adjustment of the quasi-fixed factor over time, and K_t'' denotes the long-run optimal value of the quasi-fixed factor. The static efficiency approach assumes that firms adjust to long-term optimal values instantaneously. Conversely, the dynamic efficiency approach accounts for the optimal adjustment of quasi-fixed factor over time and measures efficiency. This suggests that the static efficiency approach leads to biased estimates of efficiency measures in the dynamic production decision because it ignores the existence of adjustment costs while measuring efficiency (Chen and van Dalen 2010). Hence, when estimating accurate measures of efficiency for dynamic production decisions, one should consider the dynamic efficiency approach. In this graphical representation, the degree of efficiency is explained at a particular point in time along its adjustment path; therefore the resulting efficiency measures are temporal in nature.

The stochastic frontier model is extended to investigate long-run and short-run inefficiency levels; the resulting model is referred as a dynamic stochastic frontier approach. For instance, Ahn et al. (2000) and Tsionas (2006) suggested an autoregressive specification for the firm-specific inefficiency measures to take over the sluggish adjustments of technological innovations that relate to short- and long-run dynamics. Further, Emvalomatis (2012) proposed a stochastic production frontier model by isolating the unobserved heterogeneity from the autocorrelated inefficiency in a dynamic framework. However, these models proposed autoregressive error structure instead of integrating a structural model of sluggish adjustments and they also lacked mathematical representation of dynamic firm behavior (Emvalomatis 2009; Stefanou 2009).

Figure 1.1: Efficiency measure using dynamic linkage



Source: Adapted from Gardebroek and Oude Lansink (2008).

Some researchers have worked on the dynamic aspects of traditional production efficiency analysis, wherein they account for modeling investment decisions at the firm level, as well as measuring efficiency in a dynamic context. Analogous to static efficiency measurement, the structural approaches to modeling dynamic efficiency also use either non-parametric or parametric methods. In the case of non-parametric methods, the primal static models are extended in a dynamic framework, for instance, Nemoto and Goto (1999, 2003); Ouellette and Yan (2008) and Chen and van Dalen (2010). Furthermore, to model adjustment cost Silva and Stefanou (2003, 2007) developed a non-parametric revealed preference approach. This approach allows data to reveal the technological information, but without restricting a functional form of technology. In contrast to these methods, structural models of dynamic efficiency using parametric methods have been very scarce. For instance, Rungsuriyawiboon and Stefanou (2007) developed a structural dynamic efficiency model based on the shadow cost approach. In their model they capture how inefficiency deviates from optimal decisions rules—that is, optimal factor demand for variable and quasi-fixed factors—by constructing both actual and behavioral value functions; they have also captured all sources of inefficiency at the firm level.

The models discussed above were built on static expectations of future prices and returns, this means current prices and outputs are assumed to persist; in this situation the decision makers are unable to revise their expectations. In spite of this, these models also lack production uncertainty, but in reality farmers' make their investment decisions in an uncertain environment (Skevas et al. 2012; Skevas et al. 2014). However, the remaining challenge is how to consider uncertainty when deriving the optimal level of input allocation.

One of the simplest and most powerful theoretical methods for jointly analyzing inefficiency and production risk is the state-contingent approach (O'Donnell and Griffiths 2006; O'Donnell et al. 2010; Nauges et al. 2011; Serra et al. 2014). The results reveal that conventional (non-state contingent) models—stochastic production frontier or data envelopment analysis (DEA)—lead to biased estimates of efficiency measures when production uncertainty is ignored. However, the production risk also affects farm investment decisions (Dixit and Pindyck 1994; Pietola and Myers 2000; Serra et al. 2010; Serra et al. 2014) because risk unfolds over time and farmers have to build their expectations on costs and returns over a longer time. The empirical finding of Pietola and Myers (2000) suggests that the input price uncertainty has a negative impact on investment and it also affects optimal adjustment of the quasi-fixed factors over time, but their modeling approach works only under perfectly efficient conditions.

None of the above-mentioned dynamic models address both production uncertainty and inefficiency measures at the same time. Recently, one of the dynamic DEA models considered production uncertainty in efficiency evaluation (Skevas et al. 2012), wherein these authors introduced dynamic effects of pesticides, which has an influence on the current and future periods. These authors show that disregarding dynamic effects of production and variability in production conditions leads to an overestimation of inefficiency scores.

1.2 Objectives of the thesis

The consideration of uncertainty in the production decision is still an open question in the case of parametric dynamic efficiency models. This is because the dynamic duality theory has turned out to be a more difficult issue when dynamic production decisions merged with production uncertainty are introduced in modeling dynamic efficiency. Therefore, the advancements in the structural approaches to modeling dynamic efficiency are moving at a rather slow pace.

The main objectives of this thesis are threefold. First, the author aims to develop a dynamic efficiency model under uncertainty, an attempt that has not yet been made, thereby filling a gap in the existing literature. Here, the fundamental idea is to merge models of investment under uncertainty with (deterministic) dynamic efficiency analysis. Furthermore, the theoretical findings of the derived model are illustrated using simulations, specifically when disregarding the uncertainty in modeling dynamic efficiency. The second objective is to assess and quantify how large the omitted variable bias is on the estimates of the coefficients when uncertainty is ignored in the derived model. Third, the author explores the impact of input price and output uncertainty on optimal factor allocations in the presence of technical inefficiency (TI).

1.3 Outline

This thesis is organized as follows. Chapter 2 is devoted to the background of dynamic efficiency measurement, which is essential to the later chapters. This background entails introducing the basic concepts associated with efficiency measurement and reviewing the methodological issues related to dynamic efficiency measurements. This chapter also distinguishes between non-parametric and parametric approaches of measuring structural dynamic efficiency models. While presenting dynamic efficiency models, specific attention is given to existing models that will serve as a basis for developing the stochastic dynamic dual model of efficiency in the following chapter. Chapter 3 concentrates on theoretically deriving a dynamic efficiency model under uncertainty and empirically implementing the derived model. Following that, Chapter 4 discusses simulation ideas based on the theoretical findings, details on the design, as well as data used in simulation, and then proceeds with simulation results pertaining to omitted variable bias and the influence of uncertainty on optimal factor allocations. Chapter 5 closes with some conclusions and suggestions for further research.

2 Dynamic efficiency—state of the art

This chapter focuses primarily on the measurement of efficiency in static and dynamic settings. Section 2.1 proceeds with the basic concepts of efficiency measurement; it covers how to represent technology using sets, definitions of the production frontier, distance function and efficiency measures, as well as the formulation of a static shadow cost approach. Section 2.2 provides dynamic aspects of the production process and formulation of the dynamic model, while section 2.3 presents structural modeling approaches for measuring dynamic efficiency.

2.1 Basic concepts of efficiency measurement

2.1.1 Representing production technology

Any given technology is a black box wherein inputs are converted into outputs. There are different ways to represent the technology of a firm, but the most common way utilizes sets or functions. A production technology depicts a process by which outputs are produced from a given amount of production factors. Both input and output sets are equally essential in representing the production technology, and these sets are defined below. Definitions of a technology, an output possibilities set, an input requirements set and the distance functions are presented in this subsection based on Kumbhakar and Lovell (2000) and Coelli et al. (2005).

Definition 2.1: Let $y=\left(y_1,y_2,\ldots,y_{\overline{p}}\right)\in R_+^{\overline{p}}$ be a non-negative vector of outputs and $x=\left(x_1,x_2,\ldots,x_{\overline{n}}\right)\in R_+^{\overline{n}}$ be a non-negative vector of inputs. The production technology $GR=\{(y,x): y$ can be produced from $x\}$ explains the set of feasible input-output vectors.

Definition 2.2: The output possibilities set explains a set of output vectors that can be produced for a given level of input x, $P(x)=\{y:(y,x)\in GR\}$.

Definition 2.3: The input requirements set represents a set of input vectors that are sufficient to produce output y, $L(y)=\{x:(y,x)\in GR\}$.

These defined sets satisfy the following properties: there is no free lunch, which means zero production is possible by means of any input; these sets are closed, which ensures the presence of technically efficient input and output; these sets are also bounded, which guarantees that infinite output cannot be produced using finite input. In addition to these properties, convexity and strong disposability of inputs and outputs are placed on these sets. The definitions presented above are effective in defining and measuring efficiency based on non-parametric approaches.

A technology or production process considers either multiple inputs to produce a single output or multiple inputs to produce multiple outputs. Assuming that multiple inputs produce a single output facilitates describing the structure of a single-output production technology in terms of the production frontier. This condition applies in the rare event when a single output is produced, and also when multiple outputs are aggregated into a single output. However, the assumption of multiple-inputs to produce multiple-outputs assists in representing the structure of multiple-output production technology in terms of input and output distance functions. Both production frontiers and distance functions not only represent the structure of production technology, they are also related to measuring efficiency. In addition, the distance function plays a leading role in duality theory, which further assists in modeling static efficiency. Definitions of input and output sets are used to define the production frontier and the distance functions below.

Definition 2.4: A production frontier is a function, $f(x) = \max\{y : y \in P(x)\} = \max\{y : x \in L(y)\}$, defining a maximum output that can be produced by a known input vector.

This function provides the upper boundary of production possibilities by representing a best-practice technology of the firm. The firms operating on the production frontier are the most efficient. The distance from the input-output combination of each producer to the production frontier represents a measure of efficiency. However, this production frontier is different from the production function because the production frontier envelops the data points, whereas the production function intersects the data points.

Definition 2.5: An input distance function is defined as $D_I(y, x) = \max\{\tau : x/\tau \in L(y)\}$.

An input distance function measures the distance from a producer to the boundary of input requirements set by adopting an input-conserving approach. The function characterizes the production technology by examining a minimal proportional contraction of the input vector, given an output vector. The properties of input requirements set, such as closeness and boundedness, are necessary to attain a maximum in definition 2.5 of an input distance function. The remaining properties of the input distance function are non-decreasing in inputs, non-increasing in outputs, linearly homogenous in inputs, concave in inputs and quasi-concave in outputs. If $x \in L(y)$, then $D_I(y,x) \geq 1$, and if x belongs to the frontier of the input requirements set, then $D_I(y,x) = 1$. In other words, an input distance value of unity defines the boundary of the input requirements set to measure the technical efficiency (TE) of input use.

Definition 2.6: An output distance function is defined as $D_O(x,y) = \min\{\iota : y/\iota \in P(x)\}$.

An output distance function measures the distance from a producer to the boundary of production possibilities set by considering an output-expanding approach. The function describes the production technology by considering a maximal proportional expansion of the output vector, given an input vector. The properties of an output distance function are non-decreasing in outputs, non-increasing in inputs, linearly homogenous in outputs, quasi-convex in inputs and convex in outputs. If $y \in P(x)$, then $D_O(x,y) \leq 1$, and if y is on the frontier of the production possibilities set, then $D_O(x,y) = 1$. The output distance equal to unity defines the boundary of the production possibilities set to measure the TE of output.

2.1.2 Definitions of efficiency measures

Efficiency and productivity measures help one understand the production performance of a firm. The efficiency of a firm is measured by using a production frontier that considers only input and output quantity data, whereas firm efficiency that is measured by using cost, revenue and profit frontiers requires behavioral assumptions in addition to price and quantity data of input and/or outputs. This subsection defines the various measures of efficiency based on Kumbhakar and Lovell (2000) and Coelli et al. (2005).

Debreu (1951) and Farrell (1957) were the first to define production efficiency based on the notion of a distance function and a production possibility set, respectively. According to Farrell (1957), the TE refers to the ability of a firm to minimize its input use in the production of a given output level (input-orientated), or the ability of a firm to produce maximum output from a given input level (output-oriented).[1] Farrell also showed how to decompose economic

1 Technical inefficiency (TI) reflects the inability of a firm to produce the maximum output for given inputs (output-oriented), or it refers to an over-utilizing of inputs for a given output and input mix (input-oriented). The TI is measured as an inverse of TE, i.e., TI= 1/TE.

(overall) efficiency into technical and allocative components. This seminal work serves as a motivating factor for further developments in efficiency and productivity analysis.

Definition 2.7: An input-oriented TE is defined as $\mathrm{TE_I}(y,x)=\min\{\tau:\tau x\in L(y)\}$.

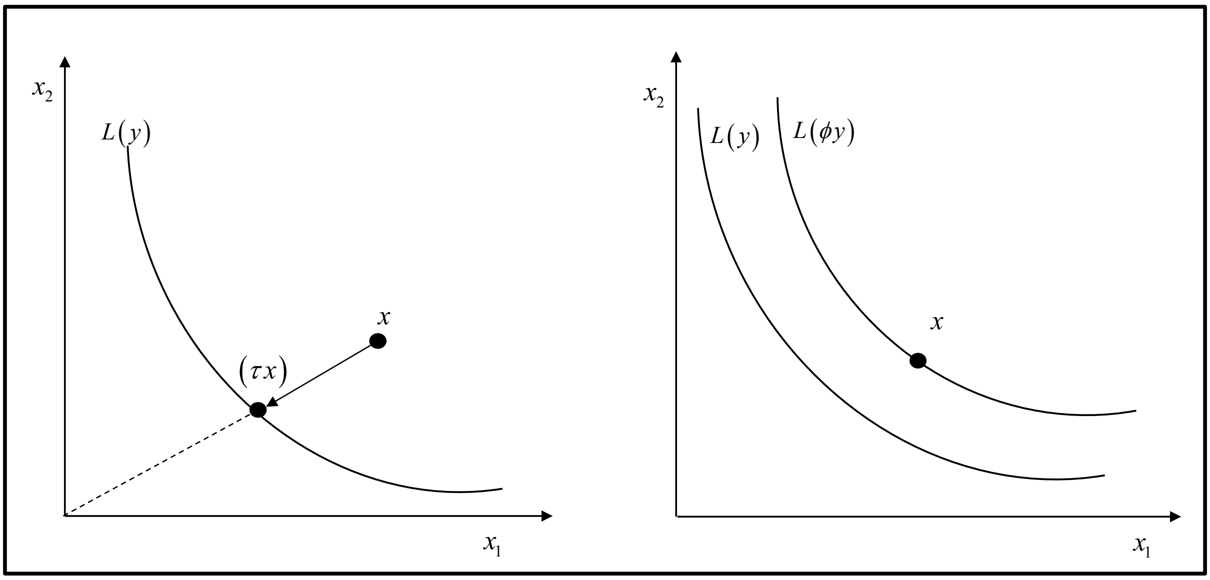
This is an input contracting TE measure and has the following properties. First, $\mathrm{TE_I}(y,x)\le 1$ means that $\mathrm{TE_I}(y,x)$ are bounded above by unity and takes values in the interval zero to unity. Second, $\mathrm{TE_I}(y,x)=1 \Leftrightarrow x\in \mathrm{Isoq}\,L(y)$, which states that the observations embedded on the production frontier are the most efficient. Third, the property of weak monotonicity, i.e., $\mathrm{TE_I}(y,x)$, is non-increasing in inputs; this says that $\mathrm{TE_I}(y,x)$ does not increase when use of any input increases. Fourth, $\mathrm{TE_I}(y,x)$ is homogeneous of degree -1 in inputs. This homogeneity property states that if there is a one-unit increase in all the inputs, a one-unit change in $\mathrm{TE_I}(y,x)$. Fifth, if the units with which output and input is measured are changed, the TE scores are unaffected.

Definition 2.8: An output-oriented TE is defined as $TE_O(x,y)=1/\left[\max\{\iota:\iota y\in P(x)\}\right]$.

This is an output-expanding TE measure and has the following properties: $\mathrm{TE_O}(x,y)\le 1$; $\mathrm{TE_O}(x,y)=1 \Leftrightarrow x\in \mathrm{Isoq}\,P(x)$; the weak monotonicity property, i.e., $\mathrm{TE_O}(x,y)$, is non-increasing in output; $\mathrm{TE_O}(x,y)$ is homogeneous of degree one in output; and the $\mathrm{TE_O}(x,y)$ score is invariant with regard to the changes in the units of any input or any output measured in the production process.

If the technology is characterized by constant returns to scale, both input-oriented and output-oriented measures of TE will provide the same results for a given input-output combination. These radial measures of TE are related to distance functions by using the following definitions: $\mathrm{TE_I}(y,x)=1/D_I(y,x)$ and $\mathrm{TE_O}(x,y)=D_O(x,y)$. Figure 2.1 illustrates the input- and output-oriented TE measures built on input set $L(y)$ and its isoquant Isoq $L(y)$. This graphical illustration is for the case of two-inputs and single-output. The input-oriented TE measures the maximal radial contraction of input x that allows sustained production of output y. The reciprocal of output-oriented TE measures the maximum expansion of output y that is achievable with available inputs x.

Figure 2.1: Input- and output-oriented measures of technical efficiency



Source: Kumbhakar and Lovell (2000)

An input-oriented measure of cost efficiency is introduced when producers' encounter with input prices $w = (w_1, w_2, \ldots, w_{\bar{n}}) \in R_{++}^{\bar{n}}$ and assume to minimize its cost in producing output y.

Definition 2.9: The measure of cost efficiency is defined as a ratio of minimum cost to observed cost, $\mathrm{CE}(y, x, w) = c(y, w) / \sum_n w_n \cdot x_n$, where n is number of variable inputs, $n = 1, 2, \ldots, \bar{n}$.

The cost efficiency fulfills the following properties. First, the cost efficiency measure is bounded between zero and one. Second, this measure is homogeneous of degree -1 in inputs, for example, a one-unit increase in all inputs increases cost by one unit and reduces cost efficiency. Third, the cost efficiency measure is non-decreasing in outputs. Fourth, this measure is homogeneous of degree zero in input prices; for instance, a one-unit increase in all input prices does not have an influence on cost efficiency.

In the above definition, $c(y, w)$ finds a solution for the cost minimization problem: $\min_x \left(\sum_n w_n \cdot x_n \right)$; subjected to the constraints $\mathrm{TE}_\mathrm{I}(y, x) \leq 1$ and given output level. The restrictions—such as the cost minimization problem and being technically efficient—need to be solved for an input-output combination to become cost efficient. If a producer fails to achieve this solution then he can opt for decomposing cost efficiency into TE and allocative efficiency (AE).[2] This decomposition is based on input-oriented approach. The notion of AE is introduced in the definition below.

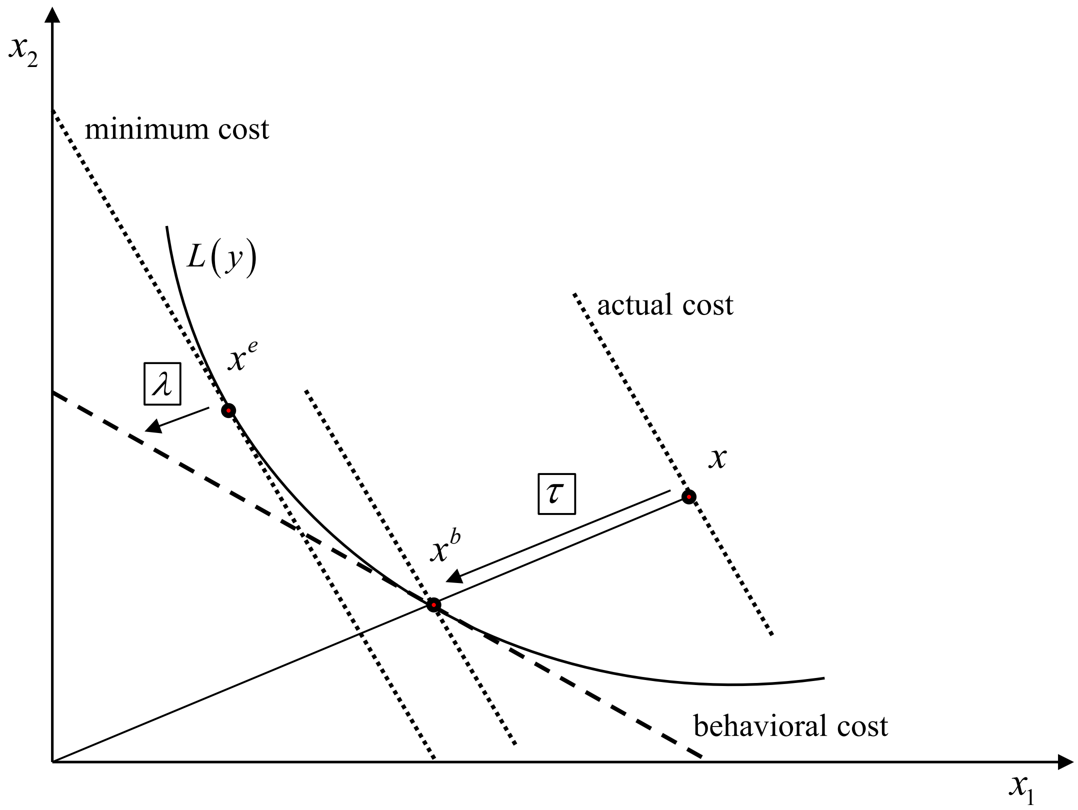
Definition 2.10: The input AE measure is defined as the ratio of cost efficiency to input-oriented TE, $\mathrm{AE_I}(y,x,w) = \mathrm{CE}(y,x,w)/\mathrm{TE_I}(y,x)$.

The input AE is a measure of the producer's adaptation to factor prices to capture the misallocation of inputs relative to their prices. The input AE measure satisfies the following properties: AE is embedded in the interval zero and unity, and is homogeneous of degree zero in input prices and quantities.

The decomposition of input-oriented cost efficiency is represented in Figure 2.2. This graphical representation is based on two inputs $(x_1$ and $x_2)$, and a single output (y). A producer can make use of inputs to produce output, which is measured using an isoquant Isoq $L(y)$. These inputs are offered at price w. The input-oriented TE is measured as: $\mathrm{TE_I}(y,x) = \tau$, i.e., the ratio of expenditure at x^b to expenditure at x, where $x^b = \tau x$. Given the market price of inputs, the cost efficiency of x is measured as the ratio of minimum cost to the actual cost. Here, x^b is technically efficient and x^e is cost efficient. Note that a distance from x to x^e consists of two parts. The first part is moved from technically inefficient x to technically efficient x^b, while the second part is moved from technically efficient x^b along the isoquant to cost efficient x^e. However, technically efficient x^b is also allocatively efficient, but with regard to shadow prices w^b. These prices are defined as the input prices, which make the technically efficient x^b a cost-minimizing solution for producing a given output, y. The conventional approach to modeling shadow prices is to scale or to translate observed prices, $w^b = \lambda w$ (cf. Kumbhakar and Lovell 2000). However, the shadow cost approach captures the deviation of market price from shadow input prices; this deviation is crucial for measuring AE, λ.

2 A firm is allocatively inefficient when it fails to allocate its inputs in the correct proportions given input prices; in other words, when the marginal rate of technical substitution between any two of its inputs is not equal to the ratio of corresponding input prices. The measure of allocative inefficiency (AI) is the inverse of AE, i.e., AI= 1/AE.

Figure 2.2: Input-orientated measure of decomposing cost efficiency



Source: Adapted from Rungsuriyawiboon and Stefanou (2007), and Kumbhakar and Lovell (2000).

2.1.3 Shadow cost approach

Decomposing economic efficiency into technical and allocative components is achieved by means of two parametric methods: the stochastic frontier approach and the shadow cost approach. The stochastic frontier approach requires restrictive functional forms and distributional assumptions to model AE. The shadow cost approach is not directly related to the frontier approach, but it is more productive due to its ease of econometric implementation. In this approach, assumptions related to firm behavior, distribution of errors and technology are relaxed when compared to frontier methods. The shadow cost approach was initially introduced by Lau and Yotopoulos (1971) and further extended by Toda (1976), and Atkinson and Halvorsen (1980). Modeling AE using the shadow cost approach uses either cost or profit functions. Atkinson and Halvorsen (1980) used a cost function to model AE and derived factor demand functions using duality theory. Atkinson and Cornwell (1994b, 1994a) revealed how to estimate AE parameters as time- and firm-specific by using panel data. The empirical application of this approach in agriculture can be found in Maietta (2000), Reinhard and Thijssen (2000) and Mosheim and Lovell (2009).

The basic idea of the shadow cost approach is that firms minimize behavioral (unobserved) costs rather than actual (observed) costs. This approach assumes that firms minimize actual costs only if the ratio of shadow prices (internal to the firm) is equivalent to the ratio of market prices. Here, input allocative inefficiency (AI) is introduced by assuming that a producer will optimize his unobserved shadow prices, which in turn parametrically relate to observed prices. In the previous section, TE is introduced as output- and input-oriented measures. In the shadow cost approach, the input-oriented measure is convenient in the framework of cost-minimization because the objective of a producer is to minimize the cost of producing a given level of output. In the case of profit-maximization, however, either an input-orientated measure or an output-orientated measure is appropriate because the objective of a producer is to allocate inputs or outputs to maximize profit.

The shadow cost approach consists of both behavioral (shadow) and actual (observed) cost functions. The shadow cost function is constructed using shadow input prices and outputs, whereas the actual cost function is constructed using market prices of inputs and outputs. Applying Shephard's Lemma to these cost functions results in input demand equations. Furthermore, the actual cost and input share equations are expressed in terms of the shadow cost function to capture how inefficiency leads to deviation from the optimal decisions. This step assists in estimating AE and TE measures to characterize a firm's behavior. A shadow cost system is estimated by specifying the functional form for the shadow cost function and appending a linear disturbance to the cost function and input share equations. In this system, the coefficient of AE appears as a parameter to be estimated, and this parameter relates to the market and shadow price of inputs. Besides, one can also compute productivity change, price elasticity of the input demand and returns to scale using a shadow cost system.

This section presents a theoretical model of AE using a shadow cost approach. Following Atkinson and Primont (2002), the behavioral cost function is given as:

$$C^b\left(y, w_n^b\right) = \min_x \left[\sum w_n^b \cdot x_n : x \in L(y)\right], \tag{2.1}$$

where $L(y)$ denotes the input requirement set, which is stated in definition 2.3. Further, x_n denotes the n^{th} variable input with $\left(x_1, \ldots, x_{\bar{n}}\right) \in \Re_+^{\bar{n}}$, y represents the output, and w_n^b is shadow (unobserved) prices with $\left(w_1^b,\ldots,w_{\bar{n}}^b\right)\in\Re_{++}^{\bar{n}}$. Since shadow prices are not observable, it makes it difficult to estimate a shadow cost function directly. For this reason, shadow prices are directly related to actual (observed) prices by inefficiency parameters, such that $w^b=\lambda_n w_n=\left(\lambda_1 w_1,\ldots,\lambda_{\bar{n}} w_{\bar{n}}\right)$, where λ_n parameters measure the deviation of observed prices from the shadow prices. The inefficiency parameters λ_n can be made time- and firm-specific by using panel data.

The cost-minimization problem in equation (2.1) is expressed in terms of a Lagrangian problem:

$$L=\left(w_n^b\cdot x_n\right)-\phi\left[D_I\left(y,x_n\right)-1\right], \tag{2.2}$$

where $D_I\left(y,x_n\right)$ denotes an input distance function and ϕ represents lagrangian multiplier. Using Shephard's Lemma, the input demand equation is given as:

$$\frac{\partial C^b\left(y,w_n^b\right)}{\partial w_n}=h_n\left(y,w_n^b\right), \qquad n=1,\ldots,\bar{n} \tag{2.3}$$

where the notation $\partial C^b\left(y,w_n^b\right)/\partial w_n$ designates the partial derivative of $C^b\left(y,w_n^b\right)$ with respect to input price w_n, and $\mathbf{h}\left(y,w_n^b\right)$ is a cost-minimizing input vector.

Using input demand equation (2.3) the actual cost function is defined as:

$$C^a=\sum_{n=1}^{\bar{n}} w_n x_n=\sum_{n=1}^{\bar{n}} w_n\cdot h_n\left(y,w_n^b\right)=\sum_{n=1}^{\bar{n}} w_n\frac{\partial C^b\left(y,w_n^b\right)}{\partial w_n}. \tag{2.4}$$

Here, $C^b\left(y,w_n^b\right)$ is linearly homogeneous in shadow price $\left(w_n^b\right)$ and by the use of Euler's theorem, the behavioral cost function is redefined as:

$$C^b\left(y,w_n^b\right)=\sum_{n=1}^{\bar{n}} w_n^b\frac{\partial C^b\left(y,w_n^b\right)}{\partial w_n}. \tag{2.5}$$

Subtracting equation (2.5) from (2.4) yields actual costs in terms of shadow prices:

$$C^a=C^b\left(y,w_n^b\right)+\sum_{n=1}^{\bar{n}}\left(1-\lambda_n\right)w_n\frac{\partial C^b\left(y,w_n^b\right)}{\partial w_n}. \tag{2.6}$$

Specifying a flexible functional form approximation to $C^b\left(y, w_n^b\right)$, one can estimate AI by appending disturbance terms to each equation. The behavioral cost system consists of the observed input demand (equation 2.3) and the observed expenditure (equation 2.6) functions. The shadow input prices in the behavioral cost system provide the dual measures of AI. This derivation serves as a basic step to generalize the static shadow cost approach in the context of dynamic dual models.

Both stochastic frontier and shadow cost approaches perform in a static framework. Both approaches have some shortcomings, however: first, there is no demarcation between inputs in the production process, that is, all inputs are treated as variable inputs; second, it ignores the explicit role of time; and third, it does not take into account how to adjust quasi-fixed factors in the long run. These shortcomings are solved in dynamic efficiency models by incorporating the cost of adjustment for quasi-fixed factors in the firm's decision problem.

2.2 Basic concepts of dynamic production decisions

As mentioned in the previous section, the static efficiency model assumes that the inputs are utilized to produce output in the same period, i.e., firms are able to adjust instantaneously (cf. Silva and Stefanou 2003). This assumption does not hold in the presence of quasi-fixed factors; hence, the static efficiency measures are biased (Nemoto and Goto 1999). The static model also ignores the intertemporal linkages in the production decisions. However, dynamic models account for intertemporal linkages in the production decisions. These intertemporal linkages may take different forms. Here, section 2.2.1 focuses on the dynamic aspects of production decisions that account for intertemporal linkages in dynamic models. Following that, section 2.2.2 formulates the dynamic dual model of an intertemporal decision problem.

2.2.1 Dynamic aspects of production decisions

This subsection focuses on the dynamic aspects of a production process that explicitly captures intertemporal effects among different periods. According to Stefanou (2009), the sources of economic dynamics are categorized as: economic forces, technological characteristics, and cognitive capacity. The economic forces include adjustment costs and financial constraint models, while the technological characteristics are comprised of physical or biological production and vintage investment. The cognitive capacity covers the learning component. In this subsection, the most important dynamic aspects—such as adjustment costs, technological innovation, and the learning component—are presented, including their interrelations when modeling dynamic models. The dynamic aspects presented here are mainly based on Stefanou (2009) and Fallah-Fini et al. (2013)

The economic forces are mainly concentrated on an adjustment process and quasi-fixed factors. In dynamic models, the source of the intertemporal linkages in production decisions is the cost of adjustment associated with changes in the quasi-fixed factor level. The adjustment cost concept was introduced by Eisner and Strotz (1963) and further extended by Lucas (1967) and Treadway (1969, 1970). Adjustment costs are classified as either external or internal. External adjustment costs appear due to market forces, whereas the internal adjustment costs arise from a reduction in productivity due to changes in quasi-fixed factor stocks (Brechling 1975). Examples of external adjustment costs include expenditures on job advertisements, expenses for initial training, architect fees, and the cost of moving new employees. In contrast, the examples of internal adjustment costs are installing new capital goods and training personnel. When modeling the dynamic theory of a firm, the external adjustment costs are added to the firm’s other costs, whereas internal adjustment costs are integrated in the technology specification of production (cf. Brechling 1975). A detailed overview on adjustment costs of quasi-fixed factors can be found in Hamermesh and Pfann (1996).

Several internal adjustment costs are observed as a learning component. This replicates the cognitive capacity to manage variations in some input levels, and also reflects managerial ability (Stefanou 2009). For instance, a manager interested in expanding his operation must assign more time to learning how to manage more resources in an effective way. In this situation, most of his time is allocated to training, for example studying manuals and attending workshops. As a result, the time spent on training is attributed to a loss of physical output since the manager allots less time to actually managing the operation. In contrast, the external adjustment costs are enforced on the decision maker. Here, due to his sluggish reaction in this situation, one cannot say that the decision maker is inefficient. The inefficiency indicates that the decision maker fails to achieve an optimality condition even though he has the ability to achieve it (Stefanou 2009). In addition, the decision maker who is inefficient might never learn from his faults. In this way, the interrelation between adjustment costs and the decision maker’s cognitive ability are explored. Many of the existing dynamic dual models account for different forms of adjustment costs due to changes in the quasi-fixed factor stock in their modeling approaches.

Learning also accumulates knowledge, and gaining additional knowledge leads to information acquisition (Stefanou 2009). Further, acquired knowledge assists in selecting the existing technologies and executing the implemented technology in the production process. In other words, knowledge supports choosing the right things to do and doing the right things in a better way. When one can translate knowledge into actions or decisions, then knowledge is translated into economic or cognitive value. Here, the open question is how to account for acquiring more knowledge and how to translate acquired knowledge into action? Further, how can one employ these issues when modeling efficiency simultaneously? To deal with competitive pressures, a decision maker attempts to find a balance between adopting innovations and exploiting the productive potential of technologies (Stefanou 2009). Here, innovation gains (necessary to keep pushing the competitive envelope), as well as efficiency gains (necessary to ensure the successful implementation of technologies), contribute to competitiveness over the long run (Stefanou 2009). However, the effective use of knowledge and learning also contributes for both sources of profitability growth—i.e., efficiency gain and innovation gain. When explaining patterns of efficiency behavior, merging technological characteristics and cognitive capacity in a structural modeling approach is still an unexplored issue.

2.2.2 Dynamic models

This subsection formulates a dynamic dual model of an intertemporal decision making problem based on Silva and Stefanou (2003); further, this formulation is connected with other dynamic dual models in the literature. In a dynamic model, the objective of a firm is to minimize the discounted sum of production costs or to maximize the discounted cash flow of net revenues over time. The behavioral assumption imposed on the firm and constraints associated with dynamic production decisions differ when departing from a static to a dynamic setting.

The objective of the firm is to minimize the discounted sum of the production costs over time. Following Silva and Stefanou (2003), the cost minimization problem is expressed in terms of a value function given as:

$$J\left(w_n, c_m, y, K_m\right) = \min_{I_m(t), x_n(t)} \int_t^{\infty} e^{-rt} \left[\sum_n \left(w_n(t) \cdot x_n(t)\right) + \sum_m \left(c_m(t) \cdot K_m(t)\right)\right] \cdot dt\,, \tag{2.7}$$

subject to

$$\left(x_n(t), I_m(t)\right) \in V\left(y(t) : K_m(t)\right)$$

$$\dot{K}_m(t) = \left(I_m(t) - \delta \cdot K_m(t)\right) \text{ and } K_0 \text{ is given,}$$

where $J(\cdot)$ denotes a long-run cost function starting at time *t*, y denotes maximum output level of the firm, x_n denotes variable inputs with $n = 1,\ldots,\overline{n}$, $\dot{K}_m$ and I_m represent net and gross investments with $m = 1,\ldots,\overline{m}$, and *c* represents the rental price of quasi-fixed factors. K_0 is the initial quasi-fixed factor stocks, r is the discount rate and δ denotes the depreciation rate. Let $V\left(y(t): K_m(t)\right)$ be the input requirement set for output, given the initial quasi-fixed factor stock. In the above dynamic optimization problem, the level of variable inputs, and the level of investment in quasi-fixed factors are considered as choice variables.

The optimization problem in equation (2.7) is subject to two constraints. The first restriction is the production constraint, which represents the technology in terms of the input requirement set. This constraint is the modified version of the input requirement set in definition 2.3 to consider adjustment costs. In the case of a parametric dynamic model, the technology representation is done in terms of the production function, wherein it accounts for the adjustment cost theory of investment. This formulation of the production function was initiated by Lucas (1967) and Treadway (1969, 1970).

The second restriction is the dynamic constraint, which is the evolution of capital stock over time. This constraint depends on the depreciation rate of existing capital and new capital investment, and operates as a linkage between existing production decisions and future production possibilities. When the adjustment cost of quasi-fixed factor stock is lacking, then the intertemporal effects, together with the dynamic constraint disappears; as a result, the real restriction for the firm is wiped out. Therefore, adjustment costs play a crucial role in the case of dynamic models because the adjustment cost concept determines a firm's optimal intertemporal behavior.

However, a solution to the optimization problem in equation (2.7) can be obtained by a Hamilton-Jacobi-Bellman equation as follows:

$$\begin{aligned} &rJ\left(w_n, c_m, y, K_m\right) = \\ &\min_{I_m(t), x_n(t)} \Bigg[\sum_n \left(w_n(t) \cdot x_n(t)\right) + \sum_m \left(c_m(t) \cdot K_m(t)\right) + \sum_m J_{K_m}\left(I_m(t) - \delta \cdot K_m(t)\right): \\ &\qquad \left(x_n(t), I_m(t)\right) \in V\left(y(t): K_m(t)\right) \Bigg], \end{aligned} \tag{2.8}$$

where $J(\cdot)$ denotes the intertemporal cost function and J_{K_m} denotes the shadow value of the m^{th} quasi-fixed factor. According to Silva and Stefanou (2007), the shadow value of a quasi-fixed factor determines the influence of a small change in the initial quasi-fixed factor stock on the value function. As a result, the shadow value of the quasi-fixed factor becomes an endogenous price and is affected by input prices, output, and initial quasi-fixed factor stock.

The empirical dynamic dual models are based on the dynamic duality theory initially proposed by McLaren and Cooper (1980) and Epstein (1981). These authors provide a dynamic duality relationship between the optimal value function and the production function for a profit maximization problem and derived dynamic factor demand functions from the value function.[3] Estimating the resulting dynamic demand functions requires flexible functional forms for the value function. For this reason, Epstein (1981) provided functional forms for the value function, which facilitates the estimation of dynamic dual models. He also mentioned that these functional forms can be made flexible by adding appropriate additional terms or parameters. The empirical applications of the dynamic dual models, which are consistent with the adjustment theory of the firm, was first given by Epstein and Denny (1983) and followed by Taylor and Monson (1985), Vasavada and Chambers (1986), Howard and Shumway (1988), and Vasavada and Ball (1988).

Most of these dynamic dual models work with an assumption of static expectations of input price and output. This means current prices and outputs are assumed to persist; in this situation the decision makers are unable to revise their expectations. However, the firm revises its production plans and output expectations as the base period changes. As a result, the firm fails to anticipate revisions in expectations. Extending the dynamic dual model with non-static price and output expectations is given by Luh and Stefanou (1996), which is further extended by Pietola and Myers (2000), Sckokai and Moro (2009), and Serra et al. (2010). All of these models are based on the assumption of perfectly efficient firms; as a result these models fail to account for the firm's inefficiency behavior.

3 The optimal decision rules—in terms of factor-demand equations—are obtained by either a primal approach through first-order conditions or a dual approach by applying envelope theorem on the intertemporal value function (Howard and Shumway 1988). The primal approach is restricted to modeling only one quasi-fixed factor, or assuming independent adjustment between two or more quasi-fixed factors. In contrast, the dual approach can model more than one quasi-fixed factor, and tests for independent adjustment instead of assuming it.

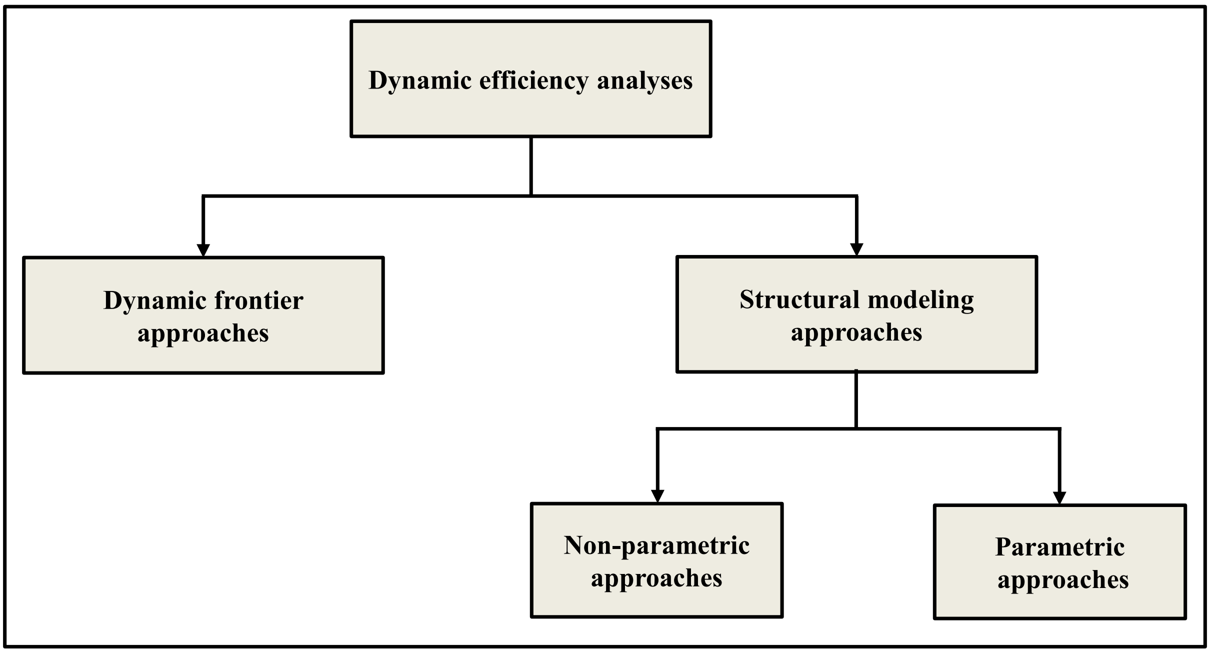
2.3 Measuring dynamic efficiency

This subsection presents methodological developments in the structural dynamic efficiency models. Section 2.3.1 provides advancements in the non-parametric dynamic efficiency models. Following that, section 2.3.2 describes the ongoing progress in parametric dynamic efficiency models based on primal and dual representations.

Most advancements in the efficiency literature have come with a static view of the firm; the static modeling of efficiency is done either using panel data or cross-section data. In the case of panel data, due to repeated observations of the firm one has to decide the true nature of efficiency. However, the usage of panel data in evaluating efficiency draws attention to the drawbacks in a static framework. The earlier panel data models are based on the time-invariant inefficiency assumption using random- and fixed-effect techniques (Schmidt and Sickles 1984). In these models, inefficiency remains constant over time when the panel is long, and also face the incidental parameter problem when the panel is short (Pitt and Lee 1981). Besides, the unobserved heterogeneity is being interpreted as inefficiency in the case of the fixed-effect model. Therefore, time-invariant inefficiency was relaxed by allowing inefficiency to vary over time. The time-varying inefficiency models were developed by three different groups (Cornwell et al. 1990; Kumbhakar 1990; Battese and Coelli 1992). In all these models, the inefficiency component of the error term explicitly depends on time. These models slightly differ with regard to the specification of error distribution, which is conditional on time. These models also impose a restriction, that is, efficiency varies over time and firm. These models only capture short-run inefficiency measures but fail to model firm-level dynamics behavior.

To characterize a firm's behavior in a dynamic setting, researchers employ both dynamic frontier approaches and structural modeling approaches. The dynamic frontier approaches are also referred to as reduced-form models because these models fail to provide mathematical representation of the dynamic behavior of the firm. On the other hand, the structural modeling approaches account for the dynamic structure of the firm's problem explicitly and employ either non-parametric or parametric methods. Figure 2.3 illustrates the different methods of measuring dynamic efficiency analysis, and these methods are detailed in the following section. In the present section the dynamic frontier approaches are not presented in detail because this thesis concentrates on the methodological improvements in the structural dynamic efficiency models.

Figure 2.3: Outline of various approaches for measuring dynamic efficiency

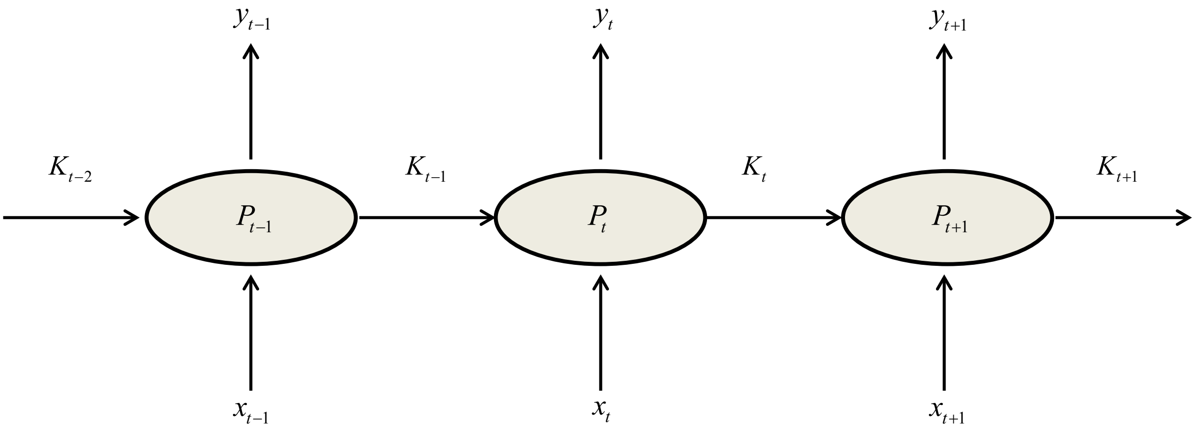


2.3.1 Non-parametric approaches

In recent years, researchers have developed dynamic models to measure efficiency by linking dynamic models of decision making with traditional efficiency analysis. These models explicitly account for time interdependence among different periods to deal with dynamic aspects of the production efficiency analysis. This subsection reviews dynamic efficiency modeling using non-parametric approaches based on a DEA model, a directional distance function, and a revealed preference approach. This subsection also presents a non-parametric dynamic efficiency model under production uncertainty. Table 2.1 presents empirical applications of the non-parametric dynamic efficiency models.

This section starts by describing dynamic efficiency models based on a conventional DEA model which is based on discrete time measure. Nemoto and Goto (1999, 2003) extended a static DEA model to a dynamic setting by integrating adjustment costs of investment and intertemporal substitution. Figure 2.4 depicts the formulation of technology in dynamic models, especially in a dynamic DEA. At the start of the period *t*, the production process P_t converts both variable inputs x_t and quasi-fixed factors K_{t-1} into output y_t, and the remaining quasi-fixed factors at the end of the period *t* are represented as K_t. This signifies that when a firm invests in quasi-fixed factors then it has to face installation costs. For installing quasi-fixed factors more resources need to be spent, and as a result fewer resources are available for the producing outputs. Note that conserving more quasi-fixed factors reallocates the current production to future periods. This is because at the starting of the period, increasing quasi-fixed factors increases output production in that period.

Figure 2.4: Dynamic technology formulation



Source: Nemoto and Goto (2003).

The modified DEA model in Nemoto and Goto (2003) determines the optimal allocation of production over time with the objective of minimizing the dynamic costs of a firm, given the technology with intertemporal substitution. These authors showed how dynamic DEA can decompose overall efficiency into static and dynamic efficiencies. In their model, the intertemporal efficient frontier of cost is given as:

$$\hat{C}(K_0) = \min_{\{x_t, K_t, \lambda_t\}_{t=1}^{T}} \left\{ \sum_{t=1}^{T} \delta\left(w_t \cdot x_t + c_t \cdot K_{t-1}\right) \middle| \left(x_t, \mathrm{K}_{t-1}, y_t, K_t\right) \in \hat{\Phi}_t \right\}, \tag{2.9}$$

where t denotes time with $t = 1, 2, \ldots, T$ and $K_0 = \bar{K}_0$, i.e., the initial values of quasi-fixed factors K_0 are given at $\bar{K}_0$. The constant discount factor is denoted by δ, x_t denotes variable inputs, K_{t-1} represents quasi-fixed factors, y_t denotes output variable, w_t is the variable input price and c_t is the quasi-fixed factor price. The frontier $\hat{C}(K_0)$ is calculated by solving the following linear programming problem:

$$\min_{\{x_t, K_t, \lambda_t\}_{t=1}^{T}} \sum_{t=1}^{T} \delta \cdot \left(w_t \cdot x_t + c_t \cdot K_{t-1}\right), \tag{2.10}$$

subject to

$$\sum_{n=1}^{\bar{n}} x_{nt} \cdot s_t \leq x_t, \qquad \mathrm{t} = 1, 2, \ldots, T$$

$$\sum_{m=1}^{\bar{m}} K_{m,t-1} \cdot s_t \leq K_{t-1}, \qquad \mathrm{t} = 1, 2, \ldots, T$$

$$\sum_{m=1}^{\bar{m}} K_{qt} \cdot s_t \geq K_t, \qquad \mathrm{t} = 1, 2, \ldots, T-1$$

$$\sum_{p=1}^{\bar{p}} y_{pt} \cdot s_t \geq y_t \qquad \mathrm{t} = 1, 2, \ldots, T$$

$$\sum_{i=1}^{N} s_{it} = 1, \qquad \mathrm{t} = 1, 2, \ldots, T$$

$$K_0 = \bar{K}_0,\ x_t \geq 0,\ K_t \geq 0,\ s_t \geq 0, \quad \mathrm{t} = 1, 2, \ldots, T,$$

where $K_{m,t-1}$ represents quasi-fixed factors at the beginning of the period *t*, K_{qt} denotes quasi-fixed factors at the end of the period *t*. The letters n, m and p represent indices for variable input, quasi-fixed factor and output, respectively. In the above-mentioned dynamic DEA model, the dynamic effects enter through adjustment costs of quasi-fixed factors.

Further, Ouellette and Yan (2008) have generalized the model developed by Nemoto and Goto (1999) to distinguish between variable inputs that can be varied in the short-run, and quasi-fixed (nondiscretionary) factors that can only be varied in the long-run. The authors incorporated intertemporal adjustment restrictions into a static cost-minimizing DEA model. These restrictions reflect an optimization over several periods, wherein a decision making unit balances the cost of an investment (acquisition costs plus adjustment costs) and the expected reduction of variable costs due to this investment. The resulting dynamic DEA allows for a decomposition of overall economic efficiency into static and dynamic efficiencies.

The process continues with a dynamic efficiency model based on the revealed preference approach. In this approach, data is used to recover technological information of the firm, but without imposing a functional form restriction on the technology. Silva and Stefanou (2003) proposed a non-parametric revealed preference approach to model adjustment costs. These authors incorporated the non-parametric dynamic dual cost approach into traditional production analysis to develop a non-parametric dynamic efficiency model. As mentioned before, these authors introduced dynamics in the production technology specification through adjustment costs with changes in the quasi-fixed factors. Here, the adjustment costs enter in terms of the properties of the family—outer and inner bounds—of the input requirement sets. The detail of their modeling approach is presented in section 2.2.2. Further, Silva and Stefanou

(2007) considered efficiency measures within the derived model. However, these efficiency measures are temporal in nature because efficiency of the firm is measured at a particular time point along its adjustment path. These authors also differentiate between short-run and long-run efficiency measures. Short-run efficiency measures only account for variable inputs in the production process, whereas long-run efficiency measures consider both variable inputs and investments in quasi-fixed factors.

The non-parametric dynamic efficiency models are built on the directional distance function. This function is a special case of input and output distance functions provided in Definition 2.5 and Definition 2.6 in section 2.1.1. Based on Silva and Stefanou's (2003) theoretical framework, the input-based dynamic efficiency model is proposed by using a directional distance function approach (Oude Lansink and Silva 2006; Silva and Oude Lansink 2009). This modeling approach is different only with respect to the production constraint specification; these authors used a directional distance function to represent the production constraint. The resulting model of dynamic directional input distance function illustrates the intertemporal duality relation between the directional input distance function and the value (dynamic cost) function. Later, Silva and Oude Lansink (2013) developed a dynamic directional input distance function within the adjustment cost framework by generalizing the directional distance function approach developed by Chambers et al. (1996) in a static context. These authors suggested that future research needs to consider the impact of technological progress and uncertainty in a dynamic production context.

One exception to the above presented non-parametric dynamic efficiency models is the consideration of production uncertainty in modeling dynamic efficiency (Skevas et al. 2012). The authors introduced dynamic effects of pesticides and production uncertainty by adjusting inefficiency scores for changes in climatic conditions into a non-parametric efficiency analysis.

2.3.2 Parametric approaches

A parametric approach of the dynamic efficiency measure utilizes econometric techniques for estimating dual and primal models. The dual models use either cost or profit functions, whereas the primal models employ either production or directional distance functions. This section proceeds with a short note on parametric modeling approaches used to measure dynamic efficiency. These parametric models are also referred to as closed-form solutions because they explicitly account for the intertemporal production decision in evaluating dynamic efficiency. Table 2.1 represents empirical applications of the parametric dynamic efficiency models.

The dynamic efficiency models are mainly developed in non-parametric settings, but are very meager in parametric settings. For instance, Rungsuriyawiboon and Stefanou (2007) developed a dynamic efficiency model, by integrating the static shadow cost approach (section 2.1.3) into the dynamic dual model of intertemporal decision making (section 2.2.2). Here, decomposing economic efficiency is achieved by a shadow cost approach wherein the authors distinguish between actual and behavioral value functions to account for how inefficiency leads to deviation from optimal decisions. The actual value function refers to the perfect efficiency condition, whereas the behavioral value function ensures the optimality condition, and these functions are associated with the price of observed and shadow input levels of the firm. In terms of value functions, the optimization problem is solved by applying a dynamic programming technique. The resulting optimal factor demand equations account for all sources of technical and allocative inefficiency with respect to variable inputs and net investment. In the presence of inefficiencies, the shadow price of the production factors will deviate from actual or market prices.

The structural dynamic efficiency model is not directly estimable; this is because these models depend on dynamic duality theory. To attain estimable factor demand functions, Rungsuriyawiboon and Stefanou (2007, 2008) suggested a quadratic functional form for the behavioral value function. The resulting system of equations consists of one optimized actual net investment demand and two optimized actual variable input demands in terms of behavioral value function. However, the dynamic efficiency model measures the firm’s inefficiency components, such as TI and AI of variable inputs and net investment, but their model lacks production uncertainty.

The aforementioned contribution to dynamic efficiency measurement is based on the assumption of static expectations of future prices and returns. This means that current prices and outputs contain all relevant information and will persist in the future. Decision makers are not allowed to anticipate revisions in their expectations. Besides, these models also disregard uncertainty in the production. To fill this gap, this thesis extends the Rungsuriyawiboon and Stefanou (2007, 2008) approach by considering uncertainty.

Table 2.1: Empirical applications of dynamic efficiency models

Author (year)	Efficiency approach	Estimation procedure	Sector, region	Variable inputs	Quasi-fixed factors	Data	Results
Non-parametric approaches							
Nemoto and Goto (2003)	Dynamic DEA	LP	Japanese electric utilities	Fuel, labor	Electricity generation plants, transmission facilities, distribution facilities	Panel of 9 utilities, 1981–95	In the case of dynamic DEA, the inefficiency is due to an inadequate intertemporal allocation of quasi-fixed factors, whereas in static DEA the inefficiency is due to disregarding the short-run fixity of quasi-fixed factors.
Silva and Stefanou (2003)	Revealed preference approach	Kernel estimation	Pennsylvania dairy operators	Hired labor, feed purchased, utilities, gas and oil, fertilizer and lime	Land, building, machinery and equipment, livestock, family labor	Balanced panel of 60 firms, 1986–92	A non-parametric approach to the dynamic theory of production does not require a functional form on the production technology. However, the weakness of the data is more easily and fully revealed in this approach than in the parametric approach.
Oude Lansink and Silva (2006)	Dynamic directional distance function	LP	Dutch glasshouse firms	Energy, materials, services	Structures, machinery and installations, labor	Balanced panel data of 89 firms, 1991–95	The decomposition of inefficiency for individual variable inputs suggests overuse of all variable inputs.
Silva and Stefanou (2007)	Revealed preference approach	LP, Kernel estimation	Pennsylvania dairy operators	Hired labor, feed purchased, utilities, gas and oil, fertilizer	Land, building, machinery and equipment, livestock, family labor	Balanced panel of 61 firms, 1986–92	Technical performance of dairy operators is superior to allocative performance.
Skevas et al. (2012)	Dynamic DEA	LP	Dutch arable farms	Fertilizers, pesticides, other variable inputs, labor, capital and land	Environmental impacts of pesticides on water organisms and biological controllers	Unbalanced panel of 188 farms, 2002–07	Disregarding the dynamic linkages—in production and effect of climatic variability in production conditions—may results in an overestimation of inefficiency scores.

Table 2.1: Empirical applications of dynamic efficiency models (cont…)

Author (year)	Efficiency approach	Estimation procedure	Sector, region	Variable inputs	Quasi-fixed factors	Data	Results
Non-parametric approaches							
Silva and Oude Lansink (2013)	Dynamic directional input distance function	LP	Dutch glasshouse firms	Energy, materials, services	Structures, installations, labor	Unbalanced panel of 103 firms, 1997–99	The TE shares the largest part of cost inefficiency. Their model also shows that the contribution of individual variable and dynamic factors of production to inefficiency
Kapelko et al. (2014)	Dynamic directional distance function		Spanish construction industry	Employee cost, material cost,	fixed assets investments	Unbalanced panel of 775 medium sized firms, 2001–09	Comparing overall mean scores of dynamic and static inefficiencies reveals that dynamic AI is lower, and technical inefficiency is higher, than static AI and TI.
Parametric approaches							
Rungsuriyawiboon and Stefanou (2007)	Dynamic shadow cost approach	ML and GMM	U.S. electric utilities	Fuel, labor and maintenance aggregate	Capital	Panel of 72 firms, 1986–99	Most of the electric utilities are under-utilized fuel and over-utilized capital inputs.
Serra et al. (2011)	Dynamic directional distance function	ML	Specialized dairy farms in Holland	Variable inputs other than feed, feed	Breeding livestock, machinery and buildings	Unbalanced panel of 639 farms, 1995–2005	Cost inefficiency is mainly due to TI, which suggests that there is a scope for cost reduction via a more efficient use of inputs.
Rungsuriyawiboon and Hockmann (2012)	Dynamic shadow cost approach	ML and GMM	Polish agricultural farms	Labor, overhead, crop inputs, livestock	Land, capital	Balanced panel of 1143 farms, 2004–07	Overuse of quasi-fixed of factors in more pronounced in the Northwest compared to Southeast region. The results also reveal sluggish adjustment process in Polish farms.

Note: DEA = Data Envelopment Analysis, ML = Maximum Likelihood, GMM = Generalized Method of Moments, LP = Linear Programming, TFP = Total factor productivity, TE = Technical Efficiency, TI = Technical Inefficiency and AI = Allocative Inefficiency.

3 Dynamic efficiency model under uncertainty

This chapter* describes how to develop theoretical and empirical model of dynamic efficiency in a stochastic setting. Section 3.1 proceeds with the general idea of the dynamic efficiency measurement with uncertainty and also provides a brief outline of a theoretical model derivation. Section 3.2 presents the derivation of a theoretical model of dynamic efficiency under uncertainty. Section 3.3 specifies the functional form for the underlying value function to derive empirical factor demand equations. Finally, section 3.4 concludes with comparative statics.

3.1 Background of dynamic efficiency under uncertainty

3.1.1 The notion of dynamic efficiency measurement with uncertainty

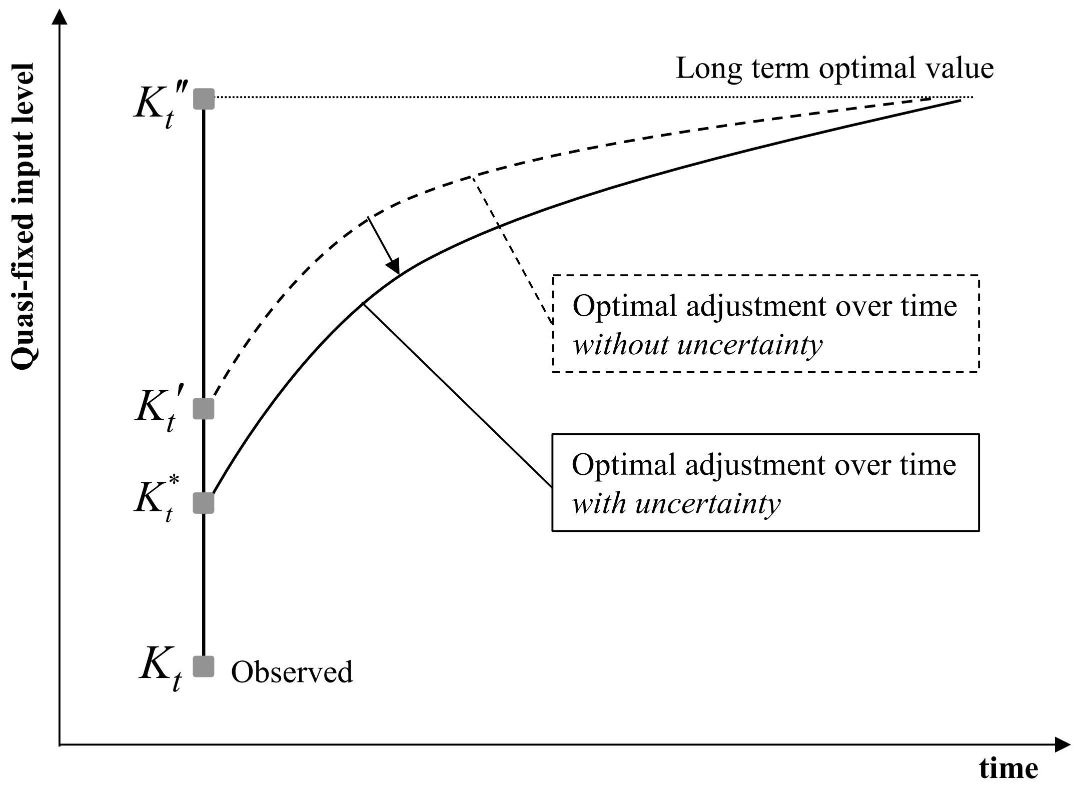
As stated in the previous chapters, the numerous static efficiency studies ignore the existence of adjustment cost and the interdependence of production decisions over time. This ignorance may cause inaccurate measures of efficiency as suggested by Gardebroek and Oude Lansink (2008). To solve this difficulty, dynamic efficiency measurement is used, which explicitly account for the interdependency of production decisions over time, adjustment costs, and also distinguishes between variable and quasi-fixed factors in the production process. However, the measurement of dynamic efficiency disregards uncertainty; this may leads to biased efficiency measures. Therefore, Figure 3.1 depicts graphical illustration of the dynamic efficiency measurement under with and without uncertainty.

In Figure 3.1, K_t is the observed level of quasi-fixed factor in a specific time *t*. The curve starting at the point K_t' represents the optimal adjustment of the quasi-fixed factor over time, and K_t'' denotes the long-term optimal value of the quasi-fixed factor. The dynamic efficiency *without uncertainty* accounts for optimal adjustment of quasi-fixed factor over time. If uncertainty is taken into account, then the optimal adjustment of the quasi-fixed factor overtime shifts downwards. As a result, the dynamic efficiency *with uncertainty* uses different reference

* This chapter is based on joint work with Silke Hüttel and Martin Odening. An earlier version is available as SiAg-Working Paper 10 in 2011, 'Measuring dynamic efficiency under uncertainty.'

point to measure efficiency. This suggests that the considering uncertainty improves the measures of dynamic efficiency.

Figure 3.1: Illustration of dynamic efficiency measurement with uncertainty



Source: Adapted from Gardebroek and Oude Lansink (2008).

3.1.2 Outline of theoretical model derivation

One of the aims of this dissertation is to develop a stochastic dynamic efficiency model by integrating the static shadow cost approach into a stochastic dual model of intertemporal decision making. A flow chart in Figure 3.2 depicts the details of the derivation procedure.

To develop such a theoretical model, the author starts with the cost-minimization problem for a representative firm, assuming that it is optimal to minimize factor inputs for a given level of output. The optimization problem is subject to the production sequence, the equation of motion for quasi-fixed factors stock and the stochastic price development over time. This decision problem is solved by using a standard procedure of stochastic dynamic programming technique, which yields optimal factor demand functions under uncertainty. The standard procedure of a stochastic dynamic dual model, however, has to be extended to capture inefficiency effects. This is achieved by means of the static shadow cost approach. The theoretical procedure is subdivided into three major steps.

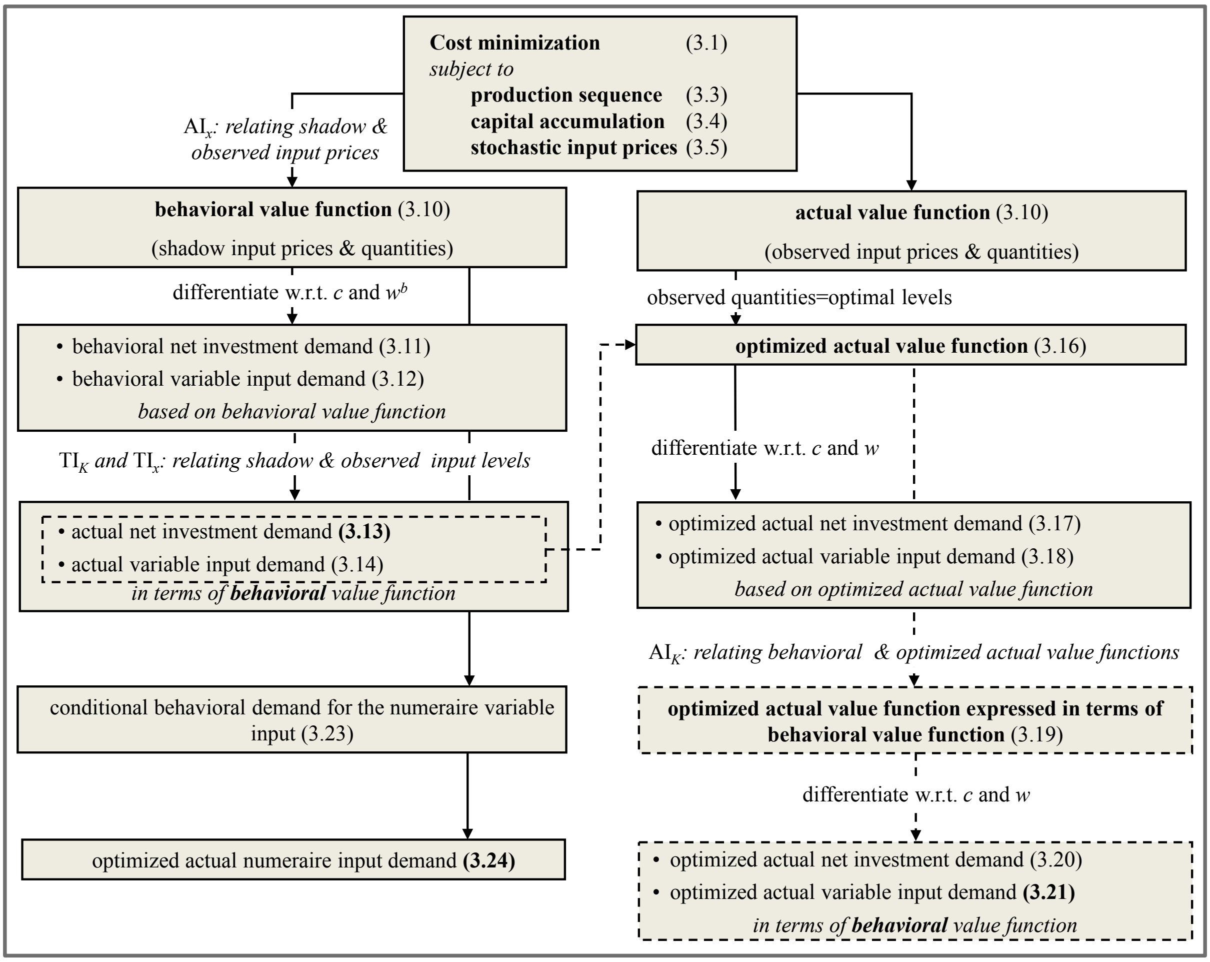
First, the *behavioral value function* (the left hand side of the flow chart in Figure 3.2) is constructed by shadow input prices and quantities. Shadow (unobserved) variable input prices are related to observed variable input prices by means of AI term. Thereby AI of variable inputs is introduced in the defined value function. The behavioral value function is differentiated with respect to shadow price of the variable input and observed price of quasi-fixed factor, which yields behavioral factor demand equations. Further, the behavioral variable and quasi-fixed factor demand functions are related to actual factor demand levels by means of TI term. Carrying out this step yields the actual quasi-fixed and variable factor demand equations in terms of the behavioral value function (the left hand side in Figure 3.2).

Second, the cost-minimization problem is solved under actual conditions. That is, the *actual value function* is constructed by observed input prices and quantities (the right hand side of the flow chart in Figure 3.2). The author assumes that the observed quantities are optimal, and replace the observed quantities by the optimized quantities in the defined actual value function. The resulting *optimized actual value function* may be interpreted as the long-run cost function. Differentiating the optimized actual value function with respect to observed input prices yields the optimized actual quasi-fixed and variable factor demands in terms of the optimized actual value function under perfectly efficient conditions.

In the third and final step, the actual factor demand equations in terms of the behavioral value function are incorporated into the optimized actual value function (dotted lines in Figure 3.2), thereby inefficiency measures are incorporated. This result in *optimized actual value function is expressed in terms of the behavioral value function* (dotted box in the right hand side of the flow chart in Figure 3.2). Differentiating this function with respect to the observed input prices and substituting the resulting derivatives in the optimized actual factor demand equations in terms of the actual value function. This result in the optimized actual factor demand equations expressed in terms of the behavioral value function (bottom of the right hand side of the flow chart in Figure 3.2). This step is necessary to identify the inefficiency parameters in terms of the actual input prices and quantities, or in other words, to measure firm inefficiency as a deviation between the optimized actual and behavioral value functions.

To satisfy a linear homogeneity restriction of the cost function, one of the variable input prices (as numeraire variable) is used to normalize the other variable input prices. The resulting AI is interpreted as price distortion in relation to the numeraire input variable to identify over- or under-use of variable input in the production process. Using the behavioral value function, the author derived the behavioral numeraire demand function for the numeraire variable input. The behavioral numeraire variable input demand is expressed in terms of the optimized actual numeraire input demand by means of TI of variable input (bottom of the left hand side of the flow chart in Figure 3.2). The numbers inside the parentheses in Figure 3.2 correspond to the equations in the derived theoretical model in the next section. The bold numbers inside the parentheses thereby indicate the final equations that serve as a base for an empirical work.

Figure 3.2: Framework of theoretical model derivation



Note: AI: Allocative Inefficiency, TI: Technical Inefficiency, c: prices of quasi-fixed factors, w: prices of variable inputs, w^b: shadow prices of variable inputs.

3.2 Theoretical model of dynamic efficiency under uncertainty

The derivation of the dynamic efficiency model under uncertainty comprises two parts: first, the static shadow cost approach for the efficiency measure (section 2.1.3); and second, a stochastic dual model of investment under uncertainty (section 3.2.1). These two parts are combined by following Rungsuriyawiboon and Stefanou's (2007) methodological procedure (section 3.2.2).

3.2.1 Cost-minimization under uncertainty

This section describes the stochastic dual model of investment under uncertainty. A representative firm minimizes its variable costs for a planned level of output, the dual variable cost function is given by (cf. Epstein and Denny 1983):

$$C_i\left(w_{ni}(t), y_i(t), K_{mi}(t), \dot{K}_{mi}(t), t\right) = \min_{x_{ni}(t)} \left\{ \sum_n \left(w_{ni}(t) \cdot x_{ni}(t) \right) \right\}, \tag{3.1}$$

where i is an index for individuals,[4] t denotes time, $x_n(t)$ denotes the level of the n^{th} variable factor with $\left[x_1(t), x_2(t), ..., x_{\bar{n}}(t)\right] \in \Re_+^{\bar{n}}$ and $\left[w_1(t), w_2(t), ..., w_{\bar{n}}(t)\right] \in \Re_{++}^{\bar{n}}$ representing the respective factor prices with $n = 1, \ldots, \bar{n}$. The variable $y(t)$ denotes the expected production level of a single output at time *t*. Further, $K_m(t)$ denotes the level of the m^{th} quasi-fixed factor with $\left[K_1(t), K_2(t), ..., K_{\bar{m}}(t)\right] \in \Re_+^{\bar{m}}$, where $m = 1, \ldots, \bar{m}$. The net investment of the respective m^{th} quasi-fixed factor is denoted by $\dot{K}_m(t)$. The variable cost function[5] in equation (3.1) reflects the least cost combination of variable input for each quantity of output $y(t)$. This function assumes that a firm takes input prices and the output level as given in the base period $t = 0$ (Epstein and Denny 1983). Here, a firm is assumed to minimize its expected discounted sum of all future cost over an infinite planning horizon, and is subject to production sequence and capital accumulation. The value function $J(\cdot)$ of the optimization problem in equation (3.1) is:

---

4 The subscript *i* (index for individuals) has been suppressed in the subsequent equations, for notational convenience.

5 The regularity conditions of the cost function are as follows (cf. Pietola and Myers 2000; Epstein and Denny 1983):

(A.1) $C \geq 0$;

(A.2) C is increasing in *y* and $\dot{K}$, and decreasing in K;

(A.3) C is convex in K and $\dot{K}$;

(A.4) C is concave in (*w*, *c*); and

(A.5) C is positively linearly homogeneous in (*w*, *c*).

$$J\left(w(0),c(0),y(0),K(0)\right)=$$
$$\min_{I_m(t)} E_0\int_0^{\infty} e^{-rt}\left[C\left(w_n(t),y(t),K_m(t),\dot{K}_m(t),t\right)+\sum_m\left(c_m(t)\cdot K_m(t)\right)\right]\cdot dt \tag{3.2}$$

where E_0 denotes the expectation operator conditional upon the information available at the present time and r represents the constant discount rate. In addition to variable input costs, the firm also incurs quasi-fixed input costs by purchasing quasi-fixed factors, which are represented by $\sum_m\left(c_m(t)\cdot K_m(t)\right)$, where $c_1(t),c_2(t),...,c_{\bar{m}}(t)$ denotes quasi-fixed factor prices. The variable $I_m(t)$ denotes gross investment in the m^{th} quasi-fixed factor.

The optimization problem in equation (3.2) is subject to production sequence as follows:

$$y_i(t)\leq F\left(x_{ni}(t),K_{mi}(t),\dot{K}_{mi}(t)\right). \tag{3.3}$$

The representative firm's technology is described by the production function $F\left(x(t),K(t),\dot{K}(t)\right)$, which defines the relationship between the size of adjustment, $\dot{K}(t)$ and the cost in terms of lost physical yield. Introducing net investment, $\dot{K}(t)$, in the production function reflects the presence of the internal cost of adjusting quasi-fixed factors in terms of the foregone output (Stefanou 1989).

The production function[6] is assumed to be concave in net investment, $\dot{K}(t)$, which implies increasing marginal adjustment costs. Adjustment in quasi-fixed factor stocks generates additional costs (i.e., positive cost of adjustment) in terms of lost production. The loss in production due to changing quasi-fixed factor stock is larger for faster adjustments; as a result, the firm tends to adjust its capital stock more slowly, such that $\dot{K}_m F_{\dot{K}_m}<0$ and $F_{\dot{K}_m\dot{K}_m}<0$ hold for all net investments (Stefanou 1989), where subscripts of F specifies partial derivatives. The existence of internal adjustment costs suggests that an increase (or decrease) in output is

6 The regularity conditions on the production function, F, are as follows (cf. Rungsuriyawiboon and Stefanou 2008):

(B.1) F is continuous and twice differentiable;

(B.2) F is finite, non-negative, real and single valued for all non-negative and finite x, K, and $\dot{K}$;

(B.3) F is strictly increasing in x and K, and is strictly concave in x; and

(B.4) F is strictly decreasing (increasing) for increasing (decreasing) in $\dot{K}$, and is strictly concave in $\dot{K}$.

associated with the contraction (or expansion) of quasi-fixed factor stock $\left(\dot{K}_m F_{\dot{K}_m} < 0\right)$. Furthermore, $F_{\dot{K}_m \dot{K}_m} < 0$ implies that the marginal cost of adjustment increases with the speed of adjustment. As a result, the sluggish behavior in adjusting the level of quasi-fixed factor is assured (Rungsuriyawiboon 2003).

The optimization problem is subject to the evolution of the quasi-fixed factors stock over time, described as follows:

$$\dot{K}_m(t) = \left(I_m(t) - \delta \cdot K_m(t)\right) \tag{3.4}$$

where δ refers to the constant depreciation rate.

In contrast to Rungsuriyawiboon and Stefanou (2007), the theoretical model in this thesis considers the non-static expectations of factor prices and output level. Therefore, the optimization problem in equation (3.2) is further subject to stochastic price development over time. In this constraint, uncertainty is introduced through a state vector, $z(t)$, which consists of the logarithms of output levels $\left(\ln y(t)\right)$, variable input prices $\left(\ln w_n(t)\right)$, and quasi-fixed factor prices $\left(\ln c_m(t)\right)$. The evolution of exogenous state variables is assumed to follow an arithmetic Brownian motion as follows:

$$dz = \alpha \cdot dt + \psi \cdot dv \tag{3.5}$$

where α denotes the vector of drift parameters, ψ is the respective variance parameter, and $\Sigma = \psi\psi'$ is a matrix that consists of the variance and co-variance parameters. In the last term, dv is the increment of a standard Wiener process with $E(dv) = 0$, $E\left[(dv)^2\right] = dt$ and $E(dv_t, dv_{t'}) = 0$ for all $t \neq t'$, where t and t' denote two different time periods.

The firm's stochastic optimization problem as given in (3.2)–(3.5) is solved by using standard stochastic dynamic programming techniques, which yields optimal decision rules in terms of the optimal value function and its derivatives (cf. Pietola and Myers 2000; Kamien and Schwartz 1991, p. 238–242). The Hamilton-Jacobi-Bellman (HJB) equation corresponding to the optimization problem in equation (3.2), is given by:

$$
\begin{aligned}
&rJ(z,\, K) = \\
&\min_{x,\, I}\left\{\sum_n \left(w_n \cdot x_n(t)\right) + \sum_m \left(c_m \cdot K_m(t)\right) + \sum_m \left(J_{K_m} \cdot \left(I_m(t) - \delta \cdot K_m(t)\right)\right)\right. \\
&\left. + \gamma(t) \cdot \left[y(t) - F\left(x_n(t), K_m(t), \dot{K}_m(t)\right)\right] + \sum_j J_{z_j} \cdot \alpha + \frac{1}{2}\Omega\right\},
\end{aligned}
\tag{3.6}
$$

where rJ is the instantaneous imputed cost of producing output, y. The partial derivative of value function, J, with respect to the m^{th} quasi-fixed factor is denoted by $J_{K_m} = \partial J / \partial K_m$. These partial derivatives can be interpreted as marginal values of the quasi-fixed factor stock or shadow values. The co-state variable $\gamma(t) = \partial J / \partial y$ is associated with the production target constraint, and is defined as the short-run marginal cost in time t (Stefanou 1989). The term $J_z = \partial J / \partial z$ denotes the partial derivatives of value function, J, with respect to the state vector, $z(t)$—includes the logarithms of factor prices and the output level. These factor prices reflect a change in the value function caused by a change in the firm's initial price level. Note that the derivative with respect to the logarithm of the output level is not explicitly interpreted since the shadow value of the output constraint is already accounted in the co-state variable γ.

The term Ω accounts for the uncertainty of input prices and the output level, and is defined as $\Omega = \sum_{j=1}^{1+\bar{n}+\bar{m}} \sum_{j'=1}^{1+\bar{n}+\bar{m}} J_{z_j z_{j'}} \sigma_{jj'}$, where J_{zz} refers to the second order partial derivatives of J with respect to state vector, $z(t)$. Indices j and j' denote the respective elements of $z(t)$, and $\sigma_{jj'}$ represents the respective variance and co-variance parameters. The optimized version of the stochastic dynamic programming in equation (3.6) states that the choice variables—variable inputs, x_n and investments, I_m—were selected to minimize the variable production costs $\sum_n \left(w_n \cdot x_n(t)\right)$, the costs of quasi-fixed factors $\sum_m \left(c_m \cdot K_m(t)\right)$, the gain from changing the stock of quasi-fixed factors $\sum_m \left(J_{K_m} \cdot \left(I_m(t) - \delta \cdot K_m(t)\right)\right)$ and the instantaneous change in the long-run cost given by $\gamma(t) \cdot \left[y(t) - F\left(x_n(t), K_m(t), \dot{K}_m(t)\right)\right]$. Note that the last two terms, $\sum_j J_{z_j} \cdot \alpha$ and $\frac{1}{2} \cdot \Omega$, arise from the stochastic evolution of the logarithms of output and input prices.

Applying Shephard's Lemma and differentiating the HJB equation (3.6) with respect to the logarithms of input prices $(\ln c_m$ and $\ln w_n)$, results in the respective conditional factor demand equations under perfect efficiency measures (Pietola and Myers 2000). The optimal net investment demand is given by:

$$\dot{K}_m^* = \left(J_{K_m,\ln c_m}\right)^{-1}\left(rJ_{\ln c_m} - c_m \cdot K_m - \sum_{m' \neq m} \dot{K}_{m'} \cdot J_{K_{m'},\ln c_m} - \alpha \cdot J_{z,\ln c_m} - \frac{1}{2}\Omega_{\ln c_m}\right), \tag{3.7}$$

where index m' indicates quasi-fixed factors other than m with $m' = 1, 2, \ldots, \overline{m} \;\; \forall m' \neq m$. Thus, according to equation (3.7), the optimal investment demand for the m^{th} quasi-fixed factor is a function of all investments in other quasi-fixed factors indicated by $\sum_{m' \neq m}\left(\dot{K}_{m'} \cdot J_{K_{m'},\ln c_m}\right)$. Furthermore, the optimal variable input demand is given as:

$$x_n^* = \frac{1}{w_n} \cdot \left(rJ_{\ln w_n} - \sum_m J_{K_m,\ln w_n} \cdot \dot{K}_m - \alpha \cdot J_{z,\ln w_n} - \frac{1}{2}\Omega_{\ln w_n}\right). \tag{3.8}$$

From equations (3.7) and (3.8) it becomes apparent that uncertain input prices will affect optimal decisions with respect to variable input and quasi-fixed factor over time.

3.2.2 Incorporating technical and allocative inefficiency

In the previous section, the optimal factor demand equations works only under perfectly efficient conditions. This situation frequently does not appear in reality because firms may be technically inefficient, allocatively inefficient, or both. Hence, inefficiency measures need to be incorporated in an above stated model. A firm's inefficiency is measured by employing the static shadow cost approach as defined by Kumbhakar and Lovell (2000). Further, Rungsuriyawiboon and Stefanou (2007) generalized this approach using the dynamic dual model of intertemporal decision making to formulate the dynamic efficiency model. This thesis employs the dynamic shadow cost approach proposed by Rungsuriyawiboon and Stefanou (2007) to integrate technical and allocative inefficiency measures in a stochastic dynamic dual model. The procedure involves three main steps.

In the first step, the *behavioral value function* J^b — superscript 'b' indicates behavioral—is defined by using shadow input prices and quantities. The behavioral value function guarantees the cost-minimizing relation under shadow prices. The HJB equation corresponding to the behavioral value function is as follows:[7]

$$
\begin{aligned}
& rJ^b\left(w_n^b(t), c_m(t), K_m(t), y(t)\right)= \\
& \sum_n\left(w_n^b(t)\cdot x_n^b(t)\right)+\sum_m\left(c_m(t)\cdot K_m(t)\right)+\sum_m\left(J_{K_m}^b\cdot\left(I_m(t)-\delta\cdot K_m(t)\right)\right) \\
& +\gamma^b(t)\cdot\left[y(t)-F\left(x_n^b(t), K_m(t), \dot{K}_m^b(t)\right)\right]+\sum_j J_{z_j}^b\cdot\alpha+\frac{1}{2}\Omega^b,
\end{aligned}
\tag{3.9}
$$

where w_n^b denotes the shadow (unobserved) prices of n^{th} variable input and are defined as $w_n^b=\lambda_n w_n=\lambda_1 w_1,\lambda_2 w_2,\ldots,\lambda_{\bar{n}} w_{\bar{n}}$. These prices are constructed to guarantee the optimal relationship, and they differ from the observed prices in the presence of allocative inefficiency measures. The symbol λ_n denotes the firm specific AI parameter for the n^{th} variable input. If $\lambda_n=1$, then the n^{th} variable input is used allocatively efficient. A value of $\lambda_n>1$ (<1) indicates that the decision maker distributes less (more) of the n^{th} input compared to the cost-minimizing allocation. The behavioral variable input demand is denoted by x_n^b and is related to the actual variable input demand $\left(x_n\right)$, such that $x_n^b=\left(1/\tau_x\right)\cdot x_n$, where $\tau_x\geq 1$ is a measure of input-oriented TI in variable input use. Similarly, $\dot{K}_m^b=\left(1/\tau_K\right)\cdot\dot{K}_m$ provides the relation between the behavioral and actual net investments of quasi-fixed factors in the presence of TI. The input-oriented measure of TI in net investment (dynamic factor) is given as $\tau_K\geq 1$.

The partial derivative of J^b with respect to K_m is denoted as $J_{K_m}^b$, is the marginal value of the behavioral quasi-fixed factor stock. This is related to the marginal value of the actual quasi-fixed factor stock, $J_{K_m}^a$, by the following definition: $J_{K_m}^b=\mu_m\cdot\left(J_{K_m}^a\right)$, where μ_m indicates the AI of net investment. The behavioral short-run marginal cost of production is represented as $\gamma^b(t)\geq 0$. The last two terms, $J_z^b\cdot\alpha$ and $\left(1/2\right)\cdot\Omega^b$, arise from the stochastic evolution of the logarithms of the output level and input prices. The stochastic term Ω^b is defined as: $\Omega^b=\sum_{j=1}^{1+\bar{n}+\bar{m}}\sum_{j'=1}^{1+\bar{n}+\bar{m}}J_{z_j z_{j'}}^b\sigma_{jj'}$, where $J_{z_j z_{j'}}^b$ denotes the second order partial derivatives of J^b with respect to z, and $\sigma_{jj'}$ represents the variance and co-variance parameters. Indices j and

7 Equation (3.9) is the optimized version of the behavioral HJB equation; therefore, the min-operator does not appear here.

j' represent the respective state variables in z. For the details of the stochastic term Ω^b see Appendix A.

To simplify the subsequent steps of model derivation, the drift rate (α) is set to zero in the arithmetic Brownian motion. The behavioral value function in equation (3.9) is rewritten as:[8]

$$rJ^b\left(\lambda_n w_n, c_m, K_m, y\right) = \\ \sum_n \left(\lambda_n w_n\right)\cdot x_n^b + \sum_m c_m \cdot K_m + \sum_m J_{K_m}^b \cdot \left(I_m - \delta \cdot K_m\right) + \gamma^b \cdot \left[y - F\left(x_n^b, K_m, \dot{K}_m^b\right)\right] + \frac{1}{2}\Omega^b. \tag{3.10}$$

Following the same steps as in the previous section to solve cost-minimization problem, that is applying Shephard's Lemma and differentiating equation (3.10) with respect to logarithm of input prices $(\ln c_m$ and $\ln \lambda_n w_n)$, yields optimal behavioral factor demands. The behavioral net investment demand function for the m^{th} quasi-fixed factor is given by:

$$\dot{K}_m^b = \frac{1}{J_{K_m,\ln c_m}^b} \cdot \left(rJ_{\ln c_m}^b - c_m \cdot K_m - \sum_{m' \neq m} \dot{K}_{m'}^b \cdot J_{K_{m'},\ln c_m}^b - \frac{1}{2}\Omega_{\ln c_m}^b \right). \tag{3.11}$$

The behavioral variable input demand for the n^{th} variable input is as follows:

$$x_n^b = \frac{1}{w_n^b} \cdot \left(rJ_{\ln w_n}^b - \sum_m J_{K_m,\ \ln w_n}^b \cdot \dot{K}_m^b - \frac{1}{2}\Omega_{\ln w_n}^b \right). \tag{3.12}$$

Using the definitions $\dot{K}_m^b = \left(1/\tau_K\right)\cdot \dot{K}_m$ and $x_n^b = \left(1/\tau_x\right)\cdot x_n$, the behavioral factor demand in equations (3.11) and (3.12) are rewritten in terms of the actual factor demand equations. The respective actual net investment demand is given by:[9]

$$\dot{K}_m = \tau_K \cdot \dot{K}_m^b = \frac{\tau_K}{J_{K_m,\ln c_m}^b} \cdot \left(rJ_{\ln c_m}^b - c_m \cdot K_m - \sum_{m' \neq m} \dot{K}_{m'}^b \cdot J_{K_{m'},\ln c_m}^b - \frac{1}{2}\cdot \Omega_{\ln c_m}^b \right) \tag{3.13}$$

and the actual variable input demand is as follows:

$$x_n = \tau_x \cdot x_n^b = \frac{\tau_x}{w_n^b} \cdot \left(rJ_{\ln w_n}^b - \sum_m J_{K_m,\ \ln w_n}^b \cdot \dot{K}_m^b - \frac{1}{2}\Omega_{\ln w_n}^b \right). \tag{3.14}$$

8 The time dependency is suppressed for notational convenience where possible.

9 Note that in the subsequent step, the observed quantities are considered as the optimal levels, and the observed quantities are replaced by the optimized observed quantities with superscript *o*.

In the second step, the *actual value function* J^a—superscript 'a' indicates actual—is constructed using observed input prices and quantities. The optimized version of the actual HJB equation corresponding to the observed input prices and quantities can be written as:

$$rJ^a = \sum_n w_n \cdot x_n + \sum_m c_m \cdot K_m + \sum_m J^a_{K_m} \cdot \dot{K}_m + \gamma^a \cdot \left[y - F\left(x_n, K_m, \dot{K}_m\right)\right] + \frac{1}{2}\Omega^a, \tag{3.15}$$

where $\Omega^a = \sum_{j=1}^{1+\bar{n}+\bar{m}} \sum_{j'=1}^{1+\bar{n}+\bar{m}} J^a_{z_j z_{j'}} \sigma_{jj'}$ represents the uncertainty term and $J^a_{z_j z_{j'}}$ denotes the second order partial derivatives of J^a with respect to z. Here, the observed input quantities considered as the optimal ones, and then the observed quantities $\left(\dot{K}_m \text{ and } x_n\right)$ are replaced by the optimized observed quantities $\left(\dot{K}^o_m \text{ and } x^o_n\right)$ in the defined actual value function. The resulting optimized actual value function represents a perfect efficiency condition. The optimized actual HJB corresponding to the optimized actual value function is given as:

$$rJ^a = \sum_n w_n \cdot x^o_n + \sum_m c_m \cdot K_m + \sum_m J^a_{K_m} \cdot \dot{K}^o_m + \frac{1}{2}\Omega^a. \tag{3.16}$$

Differentiating the optimized actual HJB equation (3.16) with respect to input prices $\left(\ln c_m \text{ and } \ln w_n\right)$ yields the optimized actual net investment demand function for the m^{th} quasi-fixed factor, is given by:

$$\dot{K}^o_m = \frac{1}{J^a_{K_m, \ln c_m}} \cdot \left(rJ^a_{\ln c_m} - c_m \cdot K_m - \sum_{m' \neq m} \dot{K}^o_{m'} \cdot J^a_{K_{m'}, \ln c_m} - \frac{1}{2}\Omega^a_{\ln c_m} \right) \tag{3.17}$$

and the optimized actual variable input demand function for the n^{th} variable input, is as follows:

$$x^o_n = \frac{1}{w_n} \cdot \left(rJ^a_{\ln w_n} - \sum_m J^a_{K_m, \ln w_n} \cdot \dot{K}^o_m - \frac{1}{2}\Omega^a_{\ln w_n} \right). \tag{3.18}$$

Since the optimized actual quantities $\left(\dot{K}^o_m \text{ and } x^o_n\right)$ represent a fully efficient input use; as a result, the optimized factor demands in equations (3.17) and (3.18) lack inefficiency measures. In other words, the optimized actual value function is equivalent to the behavioral value function in the presence of perfect efficiency, but they differ in the presence of inefficiency.

In the third and final step, the respective inefficiency measures are incorporated into to the optimized actual value function in equation (3.16), which yields the *optimized actual value function is expressed in terms of the behavioral value function*. This step is necessary to measure firm inefficiency as a deviation between the optimized actual and behavioral value functions. To achieve this, the optimized observable terms in equation (3.16) are substituted by their behavioral counter parts. That is, to introduce TI of net investment and variable inputs, $\dot{K}_m^o$ and x_n^o are substituted by equations (3.13) and (3.14), respectively. Further, the AI of net investment μ_m is introduced by using $J_{K_m}^a = \left(1/\mu_m\right) \cdot J_{K_m}^b$ and Ω^a is replaced by Ω^b.The optimized actual HJB equation is now expressed in terms of the behavioral value function, and is given as:

$$rJ^a =$$

$$\sum_n \left\{ \frac{\tau_x}{\lambda_n} \left[rJ_{\ln w_n}^b - \sum_m \left\{ \frac{J_{K_m,\ \ln w_n}^b}{J_{K_m,\ln c_m}^b} \left(rJ_{\ln c_m}^b - c_m \cdot K_m - \sum_{m' \neq m} \dot{K}_{m'}^b \cdot J_{K_{m'},\ln c_m}^b - \frac{\Omega_{\ln c_m}^b}{2} \right) \right\} \right] - \frac{\Omega_{\ln w_n}^b}{2} \right\} \qquad (3.19)$$

$$+ \sum_m c_m \cdot K_m + \sum_m \left\{ \frac{\tau_K}{\mu_m} \cdot \frac{J_{K_m}^b}{J_{K_m,\ln c_m}^b} \left(rJ_{\ln c_m}^b - c_m \cdot K_m - \sum_{m' \neq m} \dot{K}_{m'}^b \cdot J_{K_{m'},\ln c_m}^b - \frac{\Omega_{\ln c_m}^b}{2} \right) \right\} + \frac{\Omega^b}{2}.$$

The derivatives in the optimized actual net investment demand under perfect efficiency in equation (3.17), such as $J_{K_m,\ln c_m}^a$, $J_{\ln c_m}^a$, $J_{K_{m'},\ln c_m}^a$ and $\Omega_{\ln c_m}^a$, need to be expressed in terms of the behavioral value function and its derivatives. This is achieved by differentiating the optimized actual HJB equation (3.19) with respect to $\ln c_m$, $\left(K_m, \ln c_m\right)$ and $\left(K_{m'}, \ln c_m\right)$. The resulting derivatives are then inserted into equation (3.17), which yields the m^{th} optimized actual net investment demand in terms of the behavioral value function, is as follows:

$$
\begin{aligned}
&\left\{\frac{\tau_x}{r}\sum_n \frac{1}{\lambda_n}\cdot\frac{J^b_{K_m,\ln w_n}\cdot c_m}{J^b_{K_m,\ln c_m}}+\frac{1}{r}\cdot\left(1-\frac{\tau_K}{\mu_m}\right)c_m\right.\\
&\qquad+\tau_K\sum_m\frac{1}{\mu_m}\cdot\left(\frac{J^b_{\ln c_m,K_m}\cdot J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}+\frac{J^b_{K_m,K_m}\cdot J^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\right)\\
&\qquad-\frac{\tau_K}{r\mu_m}\cdot\frac{J^b_{K_m}}{J^b_{K_m,\ln c_m}}\cdot c_m-\frac{\tau_K}{r\mu_m}\cdot\frac{J^b_{K_m,K_m}}{J^b_{K_m,\ln c_m}}\cdot c_m\cdot K_m\\
&\qquad\left.-\frac{\tau_K}{2r}\sum_m\frac{1}{\mu_m}\cdot\left(\frac{\Omega^b_{\ln c_m,K_m}\cdot J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}+\frac{J^b_{K_m,K_m}\cdot\Omega^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\right)+\frac{\Omega^b_{K_m,\ln c_m}}{2r}\right\}\dot{K}^o_m=\\
&\tau_x\sum_n\frac{1}{\lambda_n}\cdot rJ^b_{\ln w_n,\ln c_m}-r\tau_x\sum_n\sum_m\frac{1}{\lambda_n}\cdot\frac{J^b_{K_m,\ln w_n}\cdot J^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\\
&+\tau_x\sum_n\frac{1}{\lambda_n}\cdot\frac{J^b_{K_m,\ln w_n}}{J^b_{K_m,\ln c_m}}\cdot c_m\cdot K_m+\frac{\tau_x}{2}\sum_n\sum_m\frac{1}{\lambda_n}\cdot\frac{J^b_{K_m,\ln w_n}\cdot\Omega^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\\
&-\frac{\tau_x}{2}\sum_n\frac{1}{\lambda_n}\cdot\Omega^b_{\ln w_n,\ln c_m}+r\tau_K\sum_m\frac{1}{\mu_m}\cdot\left(\frac{J^b_{K_m}\cdot J^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}+\frac{J^b_{\ln c_m}\cdot J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\right)\\
&-\frac{\tau_K}{\mu_m}\cdot\frac{J^b_{K_m}}{J^b_{K_m,\ln c_m}}\cdot c_m\cdot K_m-\tau_K\sum_m\frac{1}{\mu_m}\cdot\frac{J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\cdot\sum_{m'\neq m}\dot{K}^b_{m'}\cdot J^b_{K_{m'},\ln c_m}\\
&-\tau_K\sum_m\frac{1}{\mu_m}\cdot c_m\cdot K_m\cdot\frac{J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}-\frac{\tau_K}{2}\sum_m\frac{1}{\mu_m}\cdot\left(\frac{J^b_{K_m}\cdot\Omega^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}+\frac{\Omega^b_{\ln c_m}\cdot J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\right)\\
&-\tau_K\cdot\sum_{\substack{m'=1\\ \forall m'\neq m}}^{\bar{m}}\dot{K}^o_{m'}\cdot\left[\sum_m\frac{1}{\mu_m}\cdot\left(\frac{J^b_{\ln c_m,K_{m'}}\cdot J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}+\frac{J^b_{K_m,K_{m'}}\cdot J^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\right)\right.\\
&\qquad\qquad-\frac{\tau_K}{r\mu_{m'}}\cdot c_{m'}\cdot\frac{J^b_{K_{m'},\ln c_m}}{J^b_{K_{m'},\ln c_{m'}}}-\frac{\tau_K}{r\mu_m}\cdot\frac{J^b_{K_m,K_{m'}}}{J^b_{K_m,\ln c_m}}\cdot c_m\cdot K_m+\frac{\Omega^b_{K_{m'},\ln c_m}}{2r}\\
&\qquad\qquad\left.-\frac{\tau_K}{2r}\sum_m\frac{1}{\mu_m}\cdot\left(\frac{\Omega^b_{\ln c_m,K_{m'}}\cdot J^b_{K_m,\ln c_m}}{J^b_{K_m,\ln c_m}}+\frac{J^b_{K_m,K_{m'}}\cdot\Omega^b_{\ln c_m,\ln c_m}}{J^b_{K_m,\ln c_m}}\right)\right].
\end{aligned}
\tag{3.20}
$$

Note that the higher than second order derivatives of $J^b(\cdot)$ are ignored in the above equation, and the factor in use is indicated in bold to improve the readability.

The optimized actual variable input demand under perfect efficiency in equation (3.18) is expressed in terms of $J^a_{\ln w_n}$, $J^a_{K_m,\ln w_n}$ and $\Omega^a_{\ln w_n}$. Now, these terms are expressed in terms of the behavioral value function and its derivatives. This is obtained by differentiating the optimized actual HJB equation as given in equation (3.19) with respect to $\ln w_n$ and $\left(K_m, \ln w_n\right)$. The resulting derivatives are inserted into the equation (3.18), which yields the optimized actual variable input demand for the n^{th} input expressed in terms of the behavioral value function, is as follows:

$$
\begin{aligned}
x_{\boldsymbol{n}}^{o} = \frac{1}{w_{\boldsymbol{n}}} \cdot \Bigg\{ & r\tau_x \sum_n \frac{1}{\lambda_n} \cdot J^b_{\ln w_n, \ln w_{\boldsymbol{n}}} - r\tau_x \sum_n \sum_m \frac{1}{\lambda_n} \cdot \frac{J^b_{K_m,\ \ln w_n} \cdot J^b_{\ln c_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} \\
& + \frac{\tau_x}{2} \sum_n \sum_m \frac{1}{\lambda_n} \cdot \frac{J^b_{K_m,\ \ln w_n} \cdot \Omega^b_{\ln c_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} - \frac{\tau_x}{2} \sum_n \frac{1}{\lambda_n} \cdot \Omega^b_{\ln w_n, \ln w_{\boldsymbol{n}}} \\
& + r\tau_K \sum_m \frac{1}{\mu_m} \cdot \left(\frac{J^b_{K_m} \cdot J^b_{\ln c_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} + \frac{J^b_{\ln c_m} \cdot J^b_{K_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} \right) \\
& - \tau_K \sum_m \frac{1}{\mu_m} \cdot c_m \cdot K_m \cdot \frac{J^b_{K_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} - \tau_K \sum_m \frac{1}{\mu_m} \cdot \frac{J^b_{K_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} \cdot \sum_{m' \neq m} \dot{K}^b_{m'} \cdot J^b_{K_{m'}, \ln c_m} \\
& - \frac{\tau_K}{2} \sum_m \frac{1}{\mu_m} \cdot \left(\frac{J^b_{K_m} \cdot \Omega^b_{\ln c_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} + \frac{\Omega^b_{\ln c_m} \cdot J^b_{K_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} \right) \\
& - \tau_K \sum_{m''} \sum_m \dot{K}^o_{m''} \cdot \frac{1}{\mu_m} \cdot \frac{J^b_{\ln c_m, K_{m''}} \cdot J^b_{K_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} - \tau_K \sum_{m''} \sum_m \dot{K}^o_{m''} \cdot \frac{1}{\mu_m} \cdot \frac{J^b_{K_m, K_{m''}} \cdot J^b_{\ln c_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} \\
& + \frac{\tau_K}{r} \sum_m \dot{K}^o_m \cdot \frac{1}{\mu_m} \cdot \frac{c_m \cdot J^b_{K_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} + \frac{\tau_K}{2r} \sum_{m''} \sum_m \dot{K}^o_{m''} \cdot \frac{1}{\mu_m} \cdot \frac{\Omega^b_{\ln c_m, K_{m''}} \cdot J^b_{K_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} \\
& + \frac{\tau_K}{2r} \sum_{m''} \sum_m \dot{K}^o_{m''} \cdot \frac{1}{\mu_m} \cdot \frac{J^b_{K_m, K_{m''}} \cdot \Omega^b_{\ln c_m, \ln w_{\boldsymbol{n}}}}{J^b_{K_m, \ln c_m}} - \frac{1}{2r} \sum_m \Omega^b_{K_m, \ln w_{\boldsymbol{n}}} \cdot \dot{K}^o_m \Bigg\},
\end{aligned}
\tag{3.21}
$$

where $m'' = 1, \ldots, \bar{m}$. Note that in the above equation the higher than second order derivatives of $J^b(\cdot)$ are ignored, and the respective factor in use is indicated in bold to improve the readability. The variable input demand equation (3.21) is inversely related to its price $(w_{\boldsymbol{n}})$ and further a function of net investment $(\dot{K}^o_m)$, the respective level of the quasi-fixed factors (K_m) and their prices (c_m). The quadratic terms of input price volatilities $(\Omega^b_{\ln w_n, \ln w_{\boldsymbol{n}}}$ and $\Omega^b_{K_m, \ln w_{\boldsymbol{n}}})$ negatively influence the variable input demand, but the impact of remaining cross derivatives is difficult to assess a priori due to complex interaction terms. The inner arguments such as derivatives of the value function (J^b) and the Ω's are scaled interactively by the inefficiency measures—either by τ_K combined with μ_m or by τ_x together with λ_n. The dynamic factor demands equations (3.20) and (3.21) simultaneously account for both inefficiency measures and uncertainty; thereby these factor demand equations different form the deterministic factor demand equations offered by Rungsuriyawiboon and Stefanou (2007).

To ensure linear homogeneity of the cost function, input prices are normalized by one of the variable input prices (cf. Maietta 2000). Accordingly, in this thesis the first variable input price is considered as the numeraire and the shadow input prices are redefined as $w_n^b = \left(\lambda_n w_n / \lambda_1 w_1\right) = \lambda_{n1} w_{n1}$. The AI parameter, λ_{n1}, denotes the price distortion of the n^{th} variable input relative to the first variable input. A value of $\lambda_{n1} > 1$ (< 1) means that the ratio of the shadow price of the n^{th} variable input relative to the first variable input is higher (lower) than the respective observed price ratios. This implies under-use (over-use) of the n^{th} variable input in relation to the first variable input as the numeraire. Consequently, the behavioral numeraire variable input demand is derived by rearranging the optimized version of the behavioral HJB equation (3.10) as:

$$rJ^b\left(\lambda_n w_n, c_m, K_m, y\right) = x_1^b + \sum_{n=2}^{\bar{n}} w_n^b \cdot x_n^b + \sum_m c_m \cdot K_m + \sum_m J_{K_m}^b \cdot \dot{K}_m^b + \frac{1}{2}\Omega^b, \tag{3.22}$$

where x_1^b denotes the behavioral demand for the numeraire variable input, and x_n^b is the behavioral demand for the other variable inputs. The conditional behavioral demand for the numeraire variable input is expressed as:

$$x_1^b = rJ^b - \sum_{n=2}^{\bar{n}} w_n^b \cdot x_n^b - \sum_m c_m \cdot K_m - \sum_m J_{K_m}^b \cdot \dot{K}_m^b - \frac{1}{2}\Omega^b. \tag{3.23}$$

Accordingly, the optimized actual demand for the numeraire variable input is given as:

$$x_1^o = \tau_x \cdot x_1^b = \tau_x \cdot \left(rJ^b - \sum_{n=2}^{\bar{n}} w_n^b \cdot x_n^b - \sum_m c_m \cdot K_m - \sum_m J_{K_m}^b \cdot \dot{K}_m^b - \frac{1}{2}\Omega^b \right). \tag{3.24}$$

The theoretical factor demand equations (3.20), (3.21) and (3.24) serve as a base for formulating an empirical model.

3.3 Specification of value function

To derive estimable decision rules from the theoretical factor demand equations (3.20), (3.21) and (3.24) requires to select the functional form for the behavioral value function, J^b. Following the standard procedure, the value function requires fourth order derivative properties in a stochastic case to solve the duality relation between the cost and value functions. However, Pietola and Myers (2000) derived the complete characteristic properties of the value function based on the cost function properties via dynamic duality theory. Therefore, the specified functional form for the value function has to fulfill the properties defined by Pietola and Myers (2000), this will ensure that output and input price uncertainty enter into factor demand equations. The value function properties derived by Pietola and Myers (2000, for proof see p. 966) in a stochastic case are as follows:

(C.1) J is concave in (w,c),

(C.2) J_K is linear in (w,c),

(C.3) J_{zz} is linear in (w,c),

(C.4) α is non-increasing and convex in (w,c).

Following Epstein (1981) and Pietola and Myers (2000), a functional form is specified for the behavioral value function, which satisfies the four aforementioned properties:

$$\begin{aligned}
&J^b(z,K)= \\
&a_0+\sum_m b_{K_m}K_m+b_y\ln y+\sum_n b_{w_n}\ln w_n^b+\sum_m b_{c_m}\ln c_m+\frac{1}{2}\sum_m\sum_m A_{K_mK_m}\cdot(K_m)^2 \\
&+\sum_m A_{yK_m}\cdot K_m\ln y+\frac{1}{2}A_{yy}\cdot(\ln y)^2+\sum_n A_{w_ny}\cdot\ln y\ln w_n^b \qquad (3.25) \\
&+\frac{1}{2}\sum_n\sum_n A_{w_nw_n}\cdot(\ln w_n^b)^2+\sum_m A_{c_my}\cdot\ln y\ln c_m+\sum_n\sum_m A_{c_mw_n}\cdot\ln w_n^b\ln c_m \\
&+\frac{1}{2}\sum_m\sum_m A_{c_mc_m}\cdot(\ln c_m)^2+\sum_m\sum_m M_{c_mK_m}^{-1}\cdot c_mK_m+\sum_n\sum_m A_{w_nK_m}\cdot w_n^bK_m,
\end{aligned}$$

where a_0 is an unknown constant term, the *b*-parameters indicate the respective first order parameters, and the respective *A*- and *M*-parameters are second order parameters of the value function.

The concavity property of J^b with respect to input prices is achieved if the hessian matrix is negative semi-definite.[10] In contrast to non-stochastic models, the last term $\left(\sum_m \sum_m M^{-1}_{c_m K_m} \cdot c_m K_m + \sum_n \sum_m A_{w_n K_m} \cdot w^b_n K_m\right)$ in the specified behavioral value function ensures that J^b_z is quadratic, and J^b_{zz} is linear in w and c. In the previous section, α is assumed to zero; therefore, the last property of the value function (C.4) is fulfilled.

Based on the assumptions of J^b, the respective derivatives of J^b are differentiated and inserted into the optimized actual factor demand functions in equations (3.20), (3.21) and (3.24). In line with Rungsuriyawiboon and Stefanou (2007), the complex structure of the optimized actual net investment demand in equation (3.20) is replaced by equation (3.13). Therefore, the optimized factor demand equations (3.13), (3.21) and (3.24) serve as a starting point for an empirical model.

The empirical factor demand equations are derived by using the specified value function. The m^{th} optimized actual net investment demand in terms of behavioral value function as given by equation (3.13) is now specified in terms of the value function parameters:

$$
\begin{aligned}
&\dot{K}^o_{\boldsymbol{m}} = \\
&\tau_K \cdot M_{c_{\boldsymbol{m}} K_{\boldsymbol{m}}} \cdot \frac{1}{c_{\boldsymbol{m}}} \cdot \Bigg\{ r \Bigg[b_{c_{\boldsymbol{m}}} + A_{c_{\boldsymbol{m}} y} \cdot \ln y + \sum_{n=2}^{\bar{n}} A_{c_{\boldsymbol{m}} w_n} \cdot \ln w^b_n + \sum_{m=1}^{\bar{m}} A_{c_{\boldsymbol{m}} c_m} \cdot \ln c_m \\
&\qquad + \sum_{m=1}^{\bar{m}} \frac{1}{M_{c_{\boldsymbol{m}} K_m}} \cdot K_m \cdot c_{\boldsymbol{m}} \Bigg] - c_{\boldsymbol{m}} \cdot K_{\boldsymbol{m}} \\
&\qquad - \sum_{m'=1, m' \neq \boldsymbol{m}}^{\bar{m}} \frac{1}{M_{c_{\boldsymbol{m}} K_{m'}}} \cdot c_{\boldsymbol{m}} \cdot \dot{K}^b_{m'} - \frac{1}{2} \sum_{m=1}^{\bar{m}} \frac{1}{M_{c_{\boldsymbol{m}} K_m}} \cdot K_m \cdot c_{\boldsymbol{m}} \cdot \sigma^2_{\ln c_{\boldsymbol{m}}} \Bigg\},
\end{aligned}
\tag{3.26}
$$

10 The matrix notation of the specified value function is as follows:

$$
\begin{aligned}
&J^b(\mathbf{z}, \mathbf{K}) = \\
&a_0 + \begin{pmatrix} b'_K & b'_y & b'_w & b'_c \end{pmatrix} \begin{pmatrix} \mathbf{K} \\ \ln \mathbf{y} \\ \ln \mathbf{w^b} \\ \ln \mathbf{c} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \mathbf{K}' & (\ln \mathbf{y})' & (\ln \mathbf{w^b})' & (\ln \mathbf{c})' \end{pmatrix} \begin{bmatrix} A_{KK} & A'_{yK} & 0 & 0 \\ A_{yK} & A_{yy} & A'_{wy} & A'_{cy} \\ 0 & A_{wy} & A_{ww} & A'_{cw} \\ 0 & A_{cy} & A_{cw} & A_{cc} \end{bmatrix} \begin{pmatrix} \mathbf{K} \\ \ln \mathbf{y} \\ \ln \mathbf{w^b} \\ \ln \mathbf{c} \end{pmatrix} \\
&+ \mathbf{c}'\mathbf{M}^{-1}\mathbf{K} + \mathbf{w^b}'\mathbf{A}_{wK}\mathbf{K}.
\end{aligned}
$$

The zero restrictions in the matrix $\mathbf{A}$ guarantee that J^b_K is linear in the quasi-fixed and variable input prices (property C.2).

where τ_K denotes the TI parameter of net investment. Note that the factor in use is indicated in bold to improve readability. The parameter $\sigma^2_{\ln c_{\boldsymbol{m}}}$ denotes variance of the respective m^{th} quasi-fixed factor price, which enters the factor demand equation through an interaction term.

Accordingly, the n^{th} optimized actual variable input demand function using the behavioral value function (3.21) is given by:

$$
\begin{aligned}
& x_{\boldsymbol{n}}^o = \\
& \frac{1}{w_{\boldsymbol{n}}} \cdot \left\{ \frac{r\tau_x}{\lambda_{\boldsymbol{n}}} \cdot \left(A_{w_{\boldsymbol{n}} w_{\boldsymbol{n}}} + \sum_{m=1}^{\bar{m}} A_{w_{\boldsymbol{n}} K_m} K_m \cdot w_{\boldsymbol{n}}^b \right) - \frac{\tau_x}{2\lambda_{\boldsymbol{n}}} \sum_{m=1}^{\bar{m}} A_{w_{\boldsymbol{n}} K_m} K_m \cdot w_{\boldsymbol{n}}^b \cdot \sigma^2_{\ln w_{\boldsymbol{n}}} \right. \\
& \quad - r\tau_x \sum_{n=2}^{\bar{n}} \sum_{m=1}^{\bar{m}} \frac{1}{\lambda_n} \cdot \frac{M_{c_m K_m}}{c_m} \cdot A_{w_n K_m} w_n^b \cdot A_{c_m w_{\boldsymbol{n}}} + r\tau_K \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot \frac{M_{c_m K_m}}{c_m} \cdot A_{c_m w_{\boldsymbol{n}}} \cdot \\
& \quad \left(b_{K_m} + \sum_{m=1}^{\bar{m}} A_{K_m K_m} K_m + A_{y K_m} \cdot \ln y + \sum_{m=1}^{\bar{m}} \frac{c_m}{M_{c_m K_m}} + \sum_{n=2}^{\bar{n}} A_{w_n K_m} w_n^b \right) \\
& \quad + r\tau_K \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot \frac{M_{c_m K_m}}{c_m} \cdot A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b \cdot \left(b_{c_m} + A_{c_m y} \cdot \ln y + \sum_{n=2}^{\bar{n}} A_{c_m w_n} \ln w_n^b \right. \\
& \qquad \left. + \sum_{m=1}^{\bar{m}} A_{c_m c_m} \ln c_m + c_m \cdot \sum_{m=1}^{\bar{m}} \frac{K_m}{M_{c_m K_m}} \right) \\
& \quad - \tau_K \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot K_m \cdot M_{c_m K_m} \cdot A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b - \tau_K \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot M_{c_m K_m} \cdot A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b \cdot \sum_{m'=1, m' \neq m}^{\bar{m}} \frac{\dot{K}_{m'}^b}{M_{c_{m'} K_m}} \\
& \quad - \frac{\tau_K}{2} \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot \sigma^2_{\ln c_m} \cdot M_{c_m K_m} \cdot A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b \cdot \frac{K_m}{M_{c_m K_m}} \\
& \quad - \tau_K \sum_{m''=1}^{\bar{m}} \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot \left(\frac{c_m}{M_{c_m K_{m''}}} \cdot A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b + A_{K_m K_{m''}} \cdot A_{c_m w_{\boldsymbol{n}}} \right) \cdot \frac{M_{c_m K_m}}{c_m} \cdot \dot{K}_{m''}^o \\
& \quad + \frac{\tau_K}{2r} \sum_{m''=1}^{\bar{m}} \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot \frac{c_m}{M_{c_m K_{m''}}} \sigma^2_{\ln c_m} \cdot \frac{M_{c_m K_m}}{c_m} \cdot A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b \cdot \dot{K}_{m''}^o \\
& \quad \left. + \frac{\tau_K}{r} \sum_{m=1}^{\bar{m}} \frac{1}{\mu_m} \cdot M_{c_m K_m} \cdot A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b \cdot \dot{K}_m^o - \frac{1}{2r} \sum_{m=1}^{\bar{m}} A_{w_{\boldsymbol{n}} K_m} w_{\boldsymbol{n}}^b \sigma^2_{\ln w_{\boldsymbol{n}}} \cdot \dot{K}_m^o \right\},
\end{aligned}
\tag{3.27}
$$

where $m'' = 1, \ldots, \bar{m}$, τ_x is TI of variable inputs, and λ_n is AI parameter of the n^{th} variable input. The variable $\dot{K}_m^o$ denotes the optimized actual net investment demand for the m^{th} factor and $\sum_{m'=1, m' \neq m}^{\bar{m}} \dot{K}_{m'}^b$ denotes the behavioral net investment demand function for quasi-fixed factors other than *m*. The above equation (3.27) comprise of two different sources of input price uncertainty: the variance of the n^{th} variable input price $\sigma^2_{\ln w_n}$ and the variance of the m^{th} quasi-fixed factor price $\sigma^2_{\ln c_m}$.

Finally, the optimized actual demand for the numeraire variable input in equation (3.24) is expressed in terms of the value function parameters:

$$
\begin{aligned}
x_1^o &= \\
&\tau_x r \Bigg\{ a_0 + \sum_{m=1}^{\bar{m}} b_{K_m} \cdot K_m + b_y \cdot \ln y + \sum_{n=2}^{\bar{n}} b_{w_n} \cdot \ln w_n^b + \sum_{m=1}^{\bar{m}} b_{c_m} \cdot \ln c_m \\
&\quad + \frac{1}{2} \cdot \sum_{m=1}^{\bar{m}} \sum_{m=1}^{\bar{m}} K_m \cdot A_{K_m K_m} \cdot K_m + \sum_{m=1}^{\bar{m}} K_m \cdot A_{yK_m} \cdot \ln y + \frac{1}{2} \cdot \ln y \cdot A_{yy} \cdot \ln y \\
&\quad + \sum_{n=2}^{\bar{n}} \ln y \cdot A_{w_n y} \cdot \ln w_n^b + \frac{1}{2} \cdot \sum_{n=2}^{\bar{n}} \sum_{n=2}^{\bar{n}} \ln w_n^b \cdot A_{w_n w_n} \cdot \ln w_n^b + \sum_{m=1}^{\bar{m}} \ln y \cdot A_{c_m y} \cdot \ln c_m \\
&\quad + \sum_{n=2}^{\bar{n}} \sum_{m=1}^{\bar{m}} \ln w_n^b \cdot A_{c_m w_n} \cdot \ln c_m + \frac{1}{2} \cdot \sum_{m=1}^{\bar{m}} \sum_{m=1}^{\bar{m}} \ln c_m \cdot A_{c_m c_m} \cdot \ln c_m \\
&\quad + \sum_{m=1}^{\bar{m}} \sum_{m=1}^{\bar{m}} c_m \cdot \frac{1}{M_{c_m K_m}} \cdot K_m + \sum_{n=2}^{\bar{n}} \sum_{m=1}^{\bar{m}} w_n^b \cdot A_{w_n K_m} \cdot K_m \Bigg\} \\
&- \tau_x \sum_{n=2}^{\bar{n}} w_n^b \cdot \frac{x_n^o}{\tau_{x_n}} - \tau_x \sum_{m=1}^{\bar{m}} c_m \cdot K_m - \tau_x \sum_{m} \Bigg\{ \Bigg(b_{K_m} + \sum_{m=1}^{\bar{m}} A_{K_m K_m} \cdot K_m + A_{yK_m} \cdot \ln y \\
&\qquad\qquad + \sum_{m=1}^{\bar{m}} \frac{c_m}{M_{c_m K_m}} + \sum_{n=2}^{\bar{n}} A_{w_n K_m} \cdot w_n^b \Bigg) \cdot \frac{\dot{K}_m^o}{\tau_{K_m}} \Bigg\} \\
&- \frac{\tau_x}{2} \cdot \Bigg[A_{yy} \cdot \sigma^2_{\ln y} + 2 \cdot \sum_{n=2}^{\bar{n}} A_{w_n y} \cdot \sigma_{\ln w_n, \ln y} + 2 \cdot \sum_{m=1}^{\bar{m}} A_{c_m y} \cdot \sigma_{\ln c_m, \ln y} \\
&\quad + \sum_{n=2}^{\bar{n}} \Bigg(A_{w_n w_n} \cdot \sigma^2_{\ln w_n} + \sum_{m=1}^{\bar{m}} A_{w_n K_m} K_m \cdot w_n^b \cdot \sigma^2_{\ln w_n} \Bigg) + \sum_{n=2}^{\bar{n}} \sum_{m=1}^{\bar{m}} A_{c_m w_n} \cdot \sigma_{\ln c_m, \ln w_n} \\
&\quad + \sum_{m=1}^{\bar{m}} \sum_{n=2}^{\bar{n}} A_{c_m w_n} \cdot \sigma_{\ln c_m, \ln w_n} + \sum_{m=1}^{\bar{m}} \Bigg(A_{c_m c_m} \cdot \sigma^2_{\ln c_m} + \sum_{m=1}^{\bar{m}} \frac{K_m}{M_{c_m K_m}} \cdot c_m \cdot \sigma^2_{\ln c_m} \Bigg) \Bigg].
\end{aligned}
\tag{3.28}
$$

The above equation consists of both input price and output uncertainty variables through variance and co-variances. The variables $\sigma^2_{\ln y}$, $\sigma^2_{\ln w_n}$ and $\sigma^2_{\ln c_m}$ denote variances of the logarithmic output, the logarithmic variable input price and the logarithmic quasi-fixed factor price, respectively. Their respective co-variances denoted by $\sigma_{\ln w_n, \ln y}$ and $\sigma_{\ln c_m, \ln y}$. Similarly, $\sigma_{\ln c_m, \ln w_n}$ denotes the co-variance of quasi-fixed and variable input prices. Additive error terms are appended to each stochastic factor demand equation to reflect errors in the stochastic optimization.

The structure of factor demand equations (3.26), (3.27) and (3.28) differ from deterministic factor demand equations in Rungsuriyawiboon and Stefanou (2007) through the volatility dependent Ω term. Further, these stochastic factor demand equations serve as a base for simulations.

3.4 Comparative statics

The main motivation for the derivation of stochastic factor demand equations was the conjecture that uncertainty affects optimal factor demand equations, which in turn might have an impact on the estimates of a firm's (in)efficiency. The equations (3.26), (3.27) and (3.28) actually reveal the significance of factor price and output uncertainty in this context. To be specific, in the investment demand equation (3.26), the negative sign of the last term indicates that volatility in prices of quasi-fixed factors, $\sigma^2_{\ln c_m}$, reduces optimal investment (increases disinvestments). This negative sign also depends on the interaction effect of $\sigma^2_{\ln c_m}$ and $K_{\boldsymbol{m}}$. A negative relationship between investment and uncertainty was also derived by Dixit and Pindyck (1994) and empirically confirmed by Pietola and Myers (2000) and Hinrichs et al. (2008).

In the variable input demand equation (3.27), the effect of $\sigma^2_{\ln c_m}$ is apparently ambiguous, however, the effect of $\sigma^2_{\ln w_n}$ is again negative, if quasi-fixed factor level and net investment are nonnegative. In the case of disinvestments, this negative effect is dampened. The impact of uncertainty on investment and variable input demand equations can be explored by using two-way interaction effects between price uncertainties and investment, as well as price uncertainties and the quasi-fixed factor level.

Followed by, the numeraire variable input equation (3.28) comprises of variances of output and input prices, which in turn have a negative impact on the numeraire input demand equation. This effect can be either amplified or attenuated by positive or negative co-variances between stochastic variables. In addition, it is still difficult to find out the effect of different uncertainties on the numeraire input demand equation due to its complex three-way interaction effects.

Using stochastic factor demand equations (3.26)–(3.28), it is not straightforward to deduce how uncertainty will affect the parameter estimates of technical and allocative efficiency. Clearly, there is an omitted variable bias with respect to value function and inefficiency parameters if uncertainty is not included in an econometric model, but its direction and magnitude are hard to tell a priori. This bias depends on the correlation between the excluded and included variables, and on the variances of all variables (see Hanushek and Jackson 1977; Clarke 2005). However, an intuitive conjecture is that ignoring uncertainty leads to an inaccurate measure of inefficiency parameters (Skevas et al. 2012). The reason for an inaccurate measure is that actual capital stocks spuriously appear too small (or too large in the case of disinvestment) if the optimal speed of adjustment is overestimated. This means that the optimal adjustment of quasi-fixed factor over time is shifted downwards due to uncertainty, which is depicted in Figure 3.1. From this, one can conclude that the omission of uncertainty variables in the derived stochastic factor demand equations may lead to biased estimates of the model parameters.

In the case of investment demand, the TI of the quasi-fixed factor has a stronger influence compared to AI of the variable input. This effect is same for the variable input demand; in addition, the AI of quasi-fixed factor has an influence on the variable input demand. In the numeraire input demand, the TI of the variable input has more effect when compared to TI of the quasi-fixed factor along with AI of the variable input. It is also difficult to find out the individual effect of inefficiency parameters on factor demand equations because these equations are non-linear in parameters, as well as in variables. Therefore, the justifications from simulation require to investigate the interdependency of the complex parameters in equations from (3.26) to (3.28), the magnitude of the inefficiency parameters, and the complexity in the interaction effects.

4 Simulations

This chapter illustrates the theoretical findings presented in the previous chapter through Monte Carlo simulations. Section 4.1 provides the design of the simulations and defines the scenario selected in this study; section 4.2 presents the model specification used for the simulation study; section 4.3 discusses the data generation procedure; section 4.4 presents the results of the simulations. The simulation results include results of omitted variable bias on the estimates of the coefficients in the optimal factor allocations, and the impact of uncertainty on the investment and variable input demands.

The first aim of this simulation is to assess and quantify how large the omitted variable bias on the estimates of the coefficients is, and if the uncertainty variables are ignored in the optimal factor allocations. The author also quantifies this bias under varying levels of inefficiency parameters and uncertainty variables. The second aim is to explore the influence of uncertainty on optimal factor demand equations by exploring the complex interaction effects. For this, variables in the derived model are artificially generated using known parameter values. To estimate the simulated factor demands, the ordinary least squares technique is used in all scenarios. The mean of the estimates and their respective standard errors are employed to present final results. Comparing the known parameter values with the least square estimates of the coefficients from the models with and without uncertainty will reveal the extent of bias. The impact of uncertainty on optimal factor allocations can be done by exploring the interaction effect. An overview of the omitted variable bias and the multiplicative interaction effects are presented below.

The omission of relevant variables in the model specification will bias the results. This is often referred to as *omitted variable bias*. For instance, the consistency of the standard least squares estimator depends on the assumption that the explanatory variables are uncorrelated with the error term. This assumption is violated, particularly when explanatory variables are omitted from the specified (true) model. Generally, if the relevant variables are omitted from the regression model or when included variables are measured with error, this omission may lead to biased estimates of the model parameters. According to Marais and Wecker (1998), the omitted variable bias can be corrected by using the auxiliary information about unobservable measurement errors.

Researchers assume that the inclusion of the relevant variables reduced the potential danger caused by omitted variable bias. The standard omitted variable results explain the omission of a single variable or two variables in a regression (Hanushek and Jackson 1977; Clarke 2005). But these results fail to state in which situation the omitted variable need to be included in the specified model. However, the inclusion of additional relevant variables may increase or decrease the bias, but there is no evidence to support this conclusion (Clarke 2005, 2009). Therefore, Clarke (2005) mentioned the omitted variable bias as a phantom menace.

Further, Kim and Frees (2006) proposed a methodology that permits for the tests of omitted effects at single and multiple levels. Simulation results of their study showed that the omission of a variable results in the bias of both regression coefficients and variance components. The authors also suggested that omitted effects at lower levels may cause more severe bias than at higher levels. The authors identified important factors that result in bias, such as level of an omitted variable, its effect size, and sample size.

The commonly used diagnostic test to detect omitted variable bias is the regression specification error test (RESET) as proposed by Ramsey (1969). This test uses only the square of the ordinary least squares (OLS) predicted value as the test variable (Godfrey and Orme 1994). This simple test not only used in OLS regression but also used in simultaneous equation models, as well as binary panel data models (cf. Peters 2000). Some of the limitations of this test in detecting the omitted variables are also investigated by Leung and Yu (2000). However, simulations in this thesis also used the RESET test for detecting the omitted variable, and the auxiliary regression technique is employed to quantify bias.

To find out the impact of uncertainty on investment and variable input demand equations, the simple slope (marginal effect of the interested interaction term) technique suggested by Aiken and West (1991) is employed. The variables in the interaction term are mean centered, which further assists the interpretation of the *multiplicative interaction terms* (Jaccard and Turrisi 2003). When specifying the multiplicative interaction models, one should include all the constitutive terms to make valid inferences about marginal effects. Neglecting these constitutive terms fails to provide substantively meaningful estimates of the marginal effects and their standard errors (Brambor et al. 2006). In the case of multiplicative interaction models, the moderator variable is often used to test the joint effect of the two or more independent variables on a dependent variable. However, exploring the three-way interaction effect is more complicated compared to the two-way interaction effect, because calculating differences between pairs of slopes and the respective standard errors is cumbersome work in the three-way interaction effect.

4.1 Design of simulation

To quantify the omitted variable bias in the derived factor demand equations, a model with uncertainty and a model without uncertainty were selected. The model with uncertainty represents the derived model, while the model without uncertainty is considered to capture the omitted variable bias. Table 4.1 presents the outline of with and without uncertainty models used in simulations. The investment demand equation comprises of only one type of uncertainty, i.e., the variance of the quasi-fixed factor price. The variable input demand equation consists of two different sources of uncertainty: the variances of quasi-fixed and variable input prices. Numeraire input demand comprises of output and input prices uncertainty, for details see Table 4.1. Ignoring each type of uncertainty helps to find out the intensity of bias on the estimates of the coefficients in the variable input demand equations. For instance, the variable input demand equation without uncertainty variables is estimated using two different models, whereas the numeraire input demand equation without uncertainty variable is estimated using three different models (Table 4.1).

Table 4.1: Outline of models with and without uncertainty in simulations

Models with uncertainty	Models without uncertainty
Investment demand equation	
with quasi-fixed factor price uncertainty	without quasi-fixed factor price uncertainty
Variable input demand equation	
with uncertainty of quasi-fixed and variable input prices	• without quasi-fixed factor price uncertainty • without variable input price uncertainty
Numeraire input demand equation	
with uncertainty of input (quasi-fixed and variable) prices and output	• without quasi-fixed factor price uncertainty • without variable input price uncertainty • without output uncertainty

Furthermore, four scenarios were defined to quantify the bias under varying levels of inefficiency parameters and uncertainty variables, (Table 4.2). The benchmark scenario considers perfectly efficient conditions. This is achieved by assigning a value of one to each inefficiency parameter in a derived model, i.e., TI and AI parameters of net investment, variable input and their prices are set to one $(\tau_K = 1,\ \tau_x = 1,\ \mu = 1$ and $\lambda_{21} = 1)$. However, remaining scenarios are selected based on changing the levels of inefficiency parameters, as well as uncertainty variables. This in turn helps to find the effect of changes in the inefficiency parameters and uncertainty variables on the stochastic factor demand equations.

As noted in the comparative statics section, in all factor demand equations, the influence of the TI of net investment is higher than the TI parameter of variable inputs. Therefore, in scenario 1, an arbitrary value of two is assigned for the TI parameter of net investment $\tau_K = 2$. This scenario helps to evaluate the effect of uncertainty on factor demand equations in the presence of TI parameter and also to capture bias changes on the estimated coefficients.

Scenario 2 represents the change in the quasi-fixed factor price uncertainty (see Table 4.2). To achieve this, the variance of quasi-fixed factor price in the benchmark is doubled, i.e., $\left(\sigma^2_{\ln c} \cdot 2\right)$. This helps to find the effect of increase in quasi-fixed factor price uncertainty on factor allocations and also to capture the bias on the estimated coefficients.

Scenario 3 signifies the change in the variable input price uncertainty, wherein the variance of variable input price is doubled $\left(\sigma^2_{\ln w_2} \cdot 2\right)$ to analyze its effect on the variable input demand functions.[11]

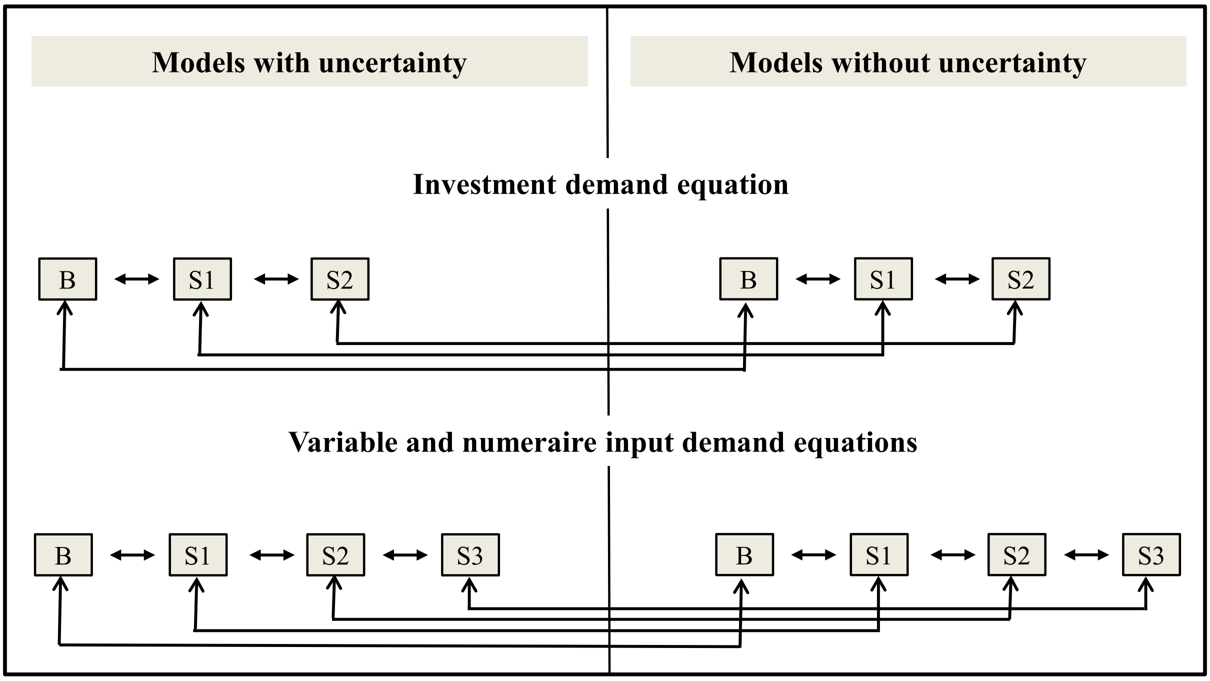
Table 4.2: Scenario settings

Scenarios	Details
Benchmark	Perfect efficiency $(\tau_K = 1,\ \tau_x = 1,\ \mu = 1$ and $\lambda_{21} = 1)$
Scenario 1	Change in the TI parameter of net investment $(\tau_K = 2)$
Scenario 2	Change in the variance of quasi-fixed factor price $\left(\sigma^2_{\ln c} \cdot 2\right)$
Scenario 3	Change in the variance of variable input price $\left(\sigma^2_{\ln w_2} \cdot 2\right)$

11 Scenarios representing the AI parameters of net investment and variable input were also considered in addition to above specified scenarios, but the results of these scenarios were similar to benchmark results. Therefore, these scenarios were dropped from simulations.

Figure 4.1 explains the comparison of four different scenarios in with and without uncertainty models of factor allocations. The estimated parameters in with and without uncertainty models are compared in each scenario. For instance, to quantify the bias in the investment demand equation, the estimates of the model parameters in scenarios (benchmark, scenario 1, and scenario 2) are compared in the model with and without uncertainty. Similarly, for variable and numeraire input demand equations each scenario (benchmark, scenario 1, scenario 2, and scenario 3) is compared with and without uncertainty models.

Figure 4.1: Comparison of different scenarios under with and without uncertainty models



Note: B is Benchmark; S1, S2 and S3 denote Scenario 1, Scenario 2 and Scenario 3, respectively.

4.2 Model specification for simulation study

The general representation of the dynamic factor demand equations from (3.26) to (3.28) is further simplified in simulations. To achieve this, one quasi-fixed factor, two variable inputs, and one output are selected in the behavioral value function in equation (3.25). Substituting $\overline{n}=2$ and $\overline{m}=1$ in equations (3.26)–(3.28) results in a simplified version of factor demand equations. The optimized actual net investment demand in terms of value function parameters is given by:

$$\begin{aligned}I &= \tau_K \cdot M_{cK} \cdot b_c \cdot r \cdot \frac{1}{c} + \tau_K \cdot M_{cK} \cdot A_{cw_2} \cdot \ln \lambda_{21} \cdot r \cdot \frac{1}{c} \\ &+ \tau_K \cdot M_{cK} \cdot A_{cy} \cdot r \cdot \frac{1}{c} \cdot \ln y + \tau_K \cdot M_{cK} \cdot A_{cw_2} \cdot r \cdot \frac{1}{c} \cdot \ln w_{21} \\ &+ \tau_K \cdot M_{cK} \cdot A_{cc} \cdot r \cdot \frac{1}{c} \cdot \ln c + \left(\tau_K \cdot r - \tau_K \cdot M_{cK} + \delta\right) \cdot K - \tau_K \cdot \frac{1}{2} \cdot \sigma^2_{\ln c} \cdot K.\end{aligned} \tag{4.1}$$

The optimized actual variable input demand equation expressed in terms of the value function parameters is give as:

$$\begin{aligned}x_2 &= \left(\frac{\tau_x}{\lambda_{21}} \cdot A_{w_2w_2} \cdot r + \frac{\tau_K}{\mu} \cdot A_{cw_2} \cdot r\right) \cdot \frac{1}{\left(w_{21}\right)} + \left(\tau_x \cdot A_{w_2K} \cdot r + \frac{\tau_K}{\mu} \cdot A_{w_2K} \cdot \lambda_{21} \cdot r\right. \\ &\left. - \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{w_2K} \cdot \lambda_{21}\right) \cdot K + \left(\frac{\tau_K}{\mu} \cdot M_{cK} \cdot b_c \cdot A_{w_2K} \cdot \lambda_{21} \cdot r - \tau_x \cdot M_{cK} \cdot A_{cw_2} \cdot A_{w_2K} \cdot r\right. \\ &\left. + \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cw_2} \cdot A_{w_2K} \cdot \lambda_{21} \cdot \ln \lambda_{21} \cdot r + \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cw_2} \cdot A_{w_2K} \cdot \lambda_{21} \cdot r\right) \cdot \frac{1}{c} \\ &+ \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cy} \cdot A_{w_2K} \cdot \lambda_{21} \cdot r \cdot \ln y \cdot \frac{1}{c} + \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cw_2} \cdot A_{w_2K} \cdot \lambda_{21} \cdot r \cdot \ln\left(w_{21}\right) \cdot \frac{1}{c} \\ &+ \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cc} \cdot A_{w_2K} \cdot \lambda_{21} \cdot r \cdot \ln c \cdot \frac{1}{c} + \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cw_2} \cdot b_K \cdot r \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} \\ &+ \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cw_2} \cdot A_{KK} \cdot r \cdot K \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} + \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{cw_2} \cdot A_{yK} \cdot r \cdot \ln y \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} \\ &- \frac{\tau_x}{2} \cdot A_{w_2K} \cdot K \cdot \sigma^2_{\ln w_2} - \frac{\tau_K}{\mu} \cdot A_{w_2K} \cdot \lambda_{21} \cdot \frac{1}{2} \cdot K \cdot \sigma^2_{\ln c} \\ &+ \left(\frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{w_2K} \cdot \lambda_{21} \cdot \frac{1}{r} - \frac{\tau_K}{\mu} \cdot A_{w_2K} \cdot \lambda_{21}\right) \cdot I - \frac{\tau_K}{\mu} \cdot M_{cK} \cdot A_{KK} \cdot A_{cw_2} \cdot I \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} \\ &+ \frac{\tau_K}{\mu} \cdot A_{w_2K} \cdot \lambda_{21} \frac{1}{2r} \cdot I \cdot \sigma^2_{\ln c} - A_{w_2K} \cdot \lambda_{21} \cdot \frac{1}{2r} \cdot I \cdot \sigma^2_{\ln w_2}.\end{aligned} \tag{4.2}$$

The optimized actual numeraire variable input demand in terms of the value function parameters is written as:

$$
\begin{aligned}
x_1 = & \left(\tau_x \cdot a_0 \cdot r + \tau_x \cdot b_{w_2} \cdot \ln \lambda_{21} \cdot r + \tau_x \cdot A_{w_2 w_2} \cdot r \cdot \frac{1}{2} \cdot \left(\ln \lambda_{21} \right)^2 \right) + \tau_x \cdot b_K \cdot r \cdot K \\
& + \tau_x \cdot A_{KK} \cdot r \cdot \frac{1}{2} \cdot K^2 + \tau_x \cdot A_{yK} \cdot r \cdot \ln y \cdot K + \tau_x \cdot A_{w_2 K} \cdot \lambda_{21} \cdot r \cdot \left(w_{21} \right) \cdot K \\
& + \tau_x \cdot \left(M_{cK} \cdot r - 1 \right) \cdot c \cdot K - \frac{\tau_x}{\tau_K} \cdot A_{KK} \cdot K \cdot I + \tau_x \cdot \left(b_y \cdot r + A_{w_2 y} \cdot \ln \lambda_{21} \cdot r \right) \cdot \ln y \\
& + \tau_x \cdot A_{yy} \cdot r \cdot \frac{1}{2} \cdot \left(\ln y \right)^2 + \tau_x \cdot A_{w_2 y} \cdot r \cdot \ln y \cdot \ln w_{21} + \tau_x \cdot A_{cy} \cdot r \cdot \ln y \cdot \ln c \\
& - \frac{\tau_x}{\tau_K} \cdot A_{yK} \cdot \ln y \cdot I + \tau_x \cdot \left(b_{w_2} \cdot r + A_{w_2 w_2} \cdot \ln \lambda_{21} \cdot r \right) \cdot \ln w_{21} + \tau_x \cdot A_{w_2 w_2} \cdot r \cdot \frac{1}{2} \cdot \left(\ln w_{21} \right)^2 \\
& + \tau_x \cdot A_{cw_2} \cdot r \cdot \ln w_{21} \cdot \ln c - \tau_x \cdot \lambda_{21} \cdot w_{21} \cdot x_2 - \frac{\tau_x}{\tau_K} \cdot A_{w_2 K} \cdot \lambda_{21} \cdot w_{21} \cdot I \\
& + \tau_x \cdot \left(A_{cw_2} \cdot \ln \lambda_{21} \cdot r + b_c \cdot r \right) \cdot \ln c + \tau_x \cdot A_{cc} \cdot r \cdot \frac{1}{2} \cdot \left(\ln c \right)^2 - \frac{\tau_x}{\tau_K} \cdot M_{cK} \cdot c \cdot I \\
& - \frac{\tau_x}{\tau_K} \cdot b_K \cdot I - \tau_x \cdot M_{cK} \cdot \frac{1}{2} \cdot K \cdot c \cdot \sigma_{\ln c}^2 - \tau_x \cdot A_{w_2 K} \cdot \lambda_{21} \cdot \frac{1}{2} \cdot K \cdot w_{21} \cdot \sigma_{\ln w_2}^2 \\
& - \tau_x \cdot A_{yy} \cdot \frac{1}{2} \cdot \sigma_{\ln y}^2 - \tau_x \cdot A_{w_2 w_2} \cdot \frac{1}{2} \cdot \sigma_{\ln w_2}^2 - \tau_x \cdot A_{cc} \cdot \frac{1}{2} \cdot \sigma_{\ln c}^2 - \tau_x \cdot A_{cw_2} \cdot \sigma_{\ln c, \ln w_2} .
\end{aligned} \tag{4.3}
$$

The derived system of factor demand equations is recursive in net investment demand, which serves as an explanatory variable in the variable and numeraire input demand equations. Furthermore, the system is recursive in variable input demand, which serves as an explanatory variable in the numeraire input demand equation.

The parameters in front of the variables are aggregated as a single coefficient in the above presented factor demand equations (4.1)–(4.3). This simplification helps to detect and quantify omitted variable bias easily. The simplified investment demand equation in (4.1) is given as:

$$
I = \theta_0 + \theta_1 \cdot \frac{1}{c} + \theta_2 \cdot \frac{1}{c} \cdot \ln y + \theta_3 \cdot \frac{1}{c} \cdot \ln \left(w_{21} \right) + \theta_4 \cdot \frac{1}{c} \cdot \ln c + \theta_5 \cdot K - \theta_6 \cdot \sigma_{\ln c}^2 \cdot K, \tag{4.4}
$$

where θ_0 represents the constant term, $\theta_1 = \left(\tau_K \cdot M_{cK} \cdot b_c \cdot r + \tau_K \cdot M_{cK} \cdot A_{cw_2} \cdot \ln \lambda_{21} \cdot r \right)$,

$\theta_2 = \tau_K \cdot M_{cK} \cdot A_{cy} \cdot r$, $\theta_3 = \tau_K \cdot M_{cK} \cdot A_{cw_2} \cdot r$, $\theta_4 = \tau_K \cdot M_{cK} \cdot A_{cc} \cdot r$,

$\theta_5 = \left(\tau_K \cdot r - \tau_K \cdot M_{cK} + \delta \right)$ and $-\theta_6 = \tau_K \cdot 0.5$.

Furthermore, in equation (4.4), the aggregated parameters in θ are calculated using the estimates of the investment demand equation. That is, τ_K is calculated by $\tau_K = -\theta_6 / 0.5$ and the remaining parameters are calculated using τ_K value: $M_{cK} = \left(-\theta_5 + \tau_K \cdot r + \delta\right)/\tau_K$, $A_{cc} = \theta_4/\left(\tau_K \cdot M_{cK} \cdot r\right)$, $A_{cw_2} = \theta_3/\left(\tau_K \cdot M_{cK} \cdot r\right)$ and $A_{cy} = \theta_2/\left(\tau_K \cdot M_{cK} \cdot r\right)$. However, parameters λ_{21} and b_c could not be identified in other scenarios except in the benchmark.

The simplified version of the variable input demand equation in (4.2) is written as:

$$
\begin{aligned}
x_2 = {} & \beta_0 + \beta_1 \cdot \frac{1}{w_{21}} + \beta_2 \cdot K + \beta_3 \cdot \frac{1}{c} + \beta_4 \cdot \ln y \cdot \frac{1}{c} + \beta_5 \cdot \ln\left(w_{21}\right) \cdot \frac{1}{c} + \beta_6 \cdot \ln c \cdot \frac{1}{c} \\
& + \beta_7 \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} + \beta_8 \cdot K \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} + \beta_9 \cdot \ln y \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} - \beta_{10} \cdot K \cdot \sigma^2_{\ln w_2} \\
& - \beta_{11} \cdot K \cdot \sigma^2_{\ln c} + \beta_{12} \cdot I - \beta_{13} \cdot I \cdot \frac{1}{w_{21}} \cdot \frac{1}{c} + \beta_{14} \cdot I \cdot \sigma^2_{\ln c} - \beta_{15} \cdot I \cdot \sigma^2_{\ln w_2}.
\end{aligned}
\tag{4.5}
$$

In the above equation, the impact of variance of quasi-fixed factor price $\left(\sigma^2_{\ln c}\right)$ on the variable input demand is not clear. Therefore, in equation (4.5), $-\beta_{11} \cdot K \cdot \sigma^2_{\ln c}$ and $+\beta_{14} \cdot I \cdot \sigma^2_{\ln c}$ terms are replaced by $\left(-\beta_{11} \cdot K + \beta_{14} \cdot I\right) \cdot \sigma^2_{\ln c}$. Using this term, one can clearly identify the net effect of the variance of the quasi-fixed factor price $\left(\sigma^2_{\ln c}\right)$ on the variable input demand equation $\left(x_2\right)$.

Subsequently, the variance of the variable input price has a negative effect on the variable input demand, given the positive values for net investment and the quasi-fixed factor stock. This can be explored by combining the common terms associated with the variance of the variable input price in equation (4.5), this results in $-\left[\beta_{10} \cdot K + \beta_{15} \cdot I\right] \cdot \sigma^2_{\ln w_2}$. This term helps to identify the bias due to input price uncertainty.

The simplified version of the numeraire variable input demand equation in (4.3) is rewritten as:

$$
\begin{aligned}
x_1 = {} & \eta_0 + \eta_1 \cdot K + \eta_2 \cdot K^2 + \eta_3 \cdot \ln y \cdot K + \eta_4 \cdot w_{21} \cdot K + \eta_5 \cdot c \cdot K - \eta_6 \cdot K \cdot I + \eta_7 \cdot \ln y \\
& + \eta_8 \cdot \left(\ln y\right)^2 + \eta_9 \cdot \ln y \cdot \ln\left(w_{21}\right) + \eta_{10} \cdot \ln y \cdot \ln c - \eta_{11} \cdot \ln y \cdot I + \eta_{12} \cdot \ln\left(w_{21}\right) \\
& + \eta_{13} \cdot \left(\ln\left(w_{21}\right)\right)^2 + \eta_{14} \cdot \ln\left(w_{21}\right) \cdot \ln c - \eta_{15} \cdot w_{21} \cdot x_2 - \eta_{16} \cdot w_{21} \cdot I + \eta_{17} \cdot \ln c \\
& + \eta_{18} \cdot \left(\ln c\right)^2 - \eta_{19} \cdot c \cdot I - \eta_{20} \cdot I - \eta_{21} \cdot K \cdot c \cdot \sigma^2_{\ln c} - \eta_{22} \cdot K \cdot w_{21} \cdot \sigma^2_{\ln w_2} \\
& - \eta_{23} \cdot \sigma^2_{\ln y} - \eta_{24} \cdot \sigma^2_{\ln w_2} - \eta_{25} \cdot \sigma^2_{\ln c},
\end{aligned}
\tag{4.6}
$$

where $\eta_0 = \tau_x \cdot a_0 \cdot r + \tau_x \cdot b_{w_2} \cdot \ln \lambda_{21} \cdot r + \tau_x \cdot A_{w_2 w_2} \cdot r \cdot \frac{1}{2} \cdot \left(\ln \lambda_{21}\right)^2$. The co-variance term is dropped in the above equation (for details see section 4.3). In equation (4.6), the variance of the quasi-fixed factor price has a negative impact on the numeraire variable input demand, given positive values of quasi-fixed factor price and quantities. To explore this effect, $-\eta_{21} \cdot K \cdot c \cdot \sigma^2_{\ln c}$ and $-\eta_{25} \cdot \sigma^2_{\ln c}$ terms are replaced by $-\left[\eta_{21} + \eta_{25}\right]\left(1 + K \cdot c\right) \cdot \sigma^2_{\ln c}$ in equation (4.6). However, the variance of the variable input price has a negative impact on the numeraire variable input demand, given the positive values of K and w. To find this effect, $-\eta_{22} \cdot K \cdot w_{21} \cdot \sigma^2_{\ln w_2}$ and $-\eta_{24} \cdot \sigma^2_{\ln w_2}$ terms in equation (4.6) are replaced by $-\left[\eta_{22} + \eta_{24}\right] \cdot \left(1 + K \cdot w_{21}\right) \cdot \sigma^2_{\ln w_2}$. This facilitates exploring the bias due to uncertainty of input prices.

In contrast to the investment demand equation, the identification of individual parameters inside β coefficients in the variable input demand in equation (4.5) and η coefficients in the numeraire input demand in equation (4.6) is complicated. Therefore, to avoid this identification problem, one should estimate the derived factor demand equations [from (4.1) to (4.3)] by using non-linear regression techniques.

4.2.1 Model specification for quantifying omitted variable bias

To quantify the omitted variable bias in the investment demand equation, models with and without uncertainty are selected. The investment demand equation with uncertainty is given by:

$$\begin{aligned} I_{it} = {} & \theta_0 + \theta_1 \cdot \frac{1}{c_t} + \theta_2 \cdot \frac{1}{c_t} \cdot \ln y_{it} + \theta_3 \cdot \frac{1}{c_t} \cdot \ln\left(w_{21}\right)_t + \theta_4 \cdot \frac{1}{c_t} \cdot \ln c_t \\ & + \theta_5 \cdot K_{it-1} - \theta_6 \cdot \left(\sigma^2_{\ln c}\right)_t \cdot K_{it-1} + \varepsilon_{I_{it}}. \end{aligned} \tag{4.7}$$

The investment demand equation without uncertainty (the variance of the quasi-fixed factor price) is given as:

$$I_{it} = \kappa_0 + \kappa_1 \cdot \frac{1}{c_t} + \kappa_2 \cdot \frac{1}{c_t} \cdot \ln y_{it} + \kappa_3 \cdot \frac{1}{c_t} \cdot \ln\left(w_{21}\right)_t + \kappa_4 \cdot \frac{1}{c_t} \cdot \ln c_t + \kappa_5 \cdot K_{it-1} + \varepsilon^*_{I_{it}}, \tag{4.8}$$

where $\varepsilon^*_{I_{it}} = -\theta_6 \cdot K_{it-1} \cdot \left(\sigma^2_{\ln c}\right)_t + \varepsilon_{I_{it}}$.

The auxiliary regression technique is used to quantify the bias on the estimates of coefficients (cf. Brambor et al. 2006). Therefore, the auxiliary regression of the quasi-fixed factor price uncertainty is written as:

$$\left[\left(\sigma^2_{\ln c}\right)_t \cdot K_{it-1}\right] = v_0 + v_1 \cdot \frac{1}{c_t} + v_2 \cdot \frac{1}{c_t} \cdot \ln y_{it} + v_3 \cdot \frac{1}{c_t} \cdot \ln\left(w_{21}\right)_t + v_4 \cdot \frac{1}{c_t} \cdot \ln c_t + v_5 \cdot K_{it-1} + e_{it}. \quad (4.9)$$

Using the results of these three regression equations, one can assess how large is the bias on the estimates of the model parameters. The calculation of bias on the estimates of the coefficients in investment demand is provided below:

$$\begin{aligned}
\kappa_0 - \theta_0 &= -\theta_6 \cdot v_0 \\
\kappa_1 - \theta_1 &= -\theta_6 \cdot v_1 \\
\kappa_2 - \theta_2 &= -\theta_6 \cdot v_2 \\
\kappa_3 - \theta_3 &= -\theta_6 \cdot v_3 \\
\kappa_4 - \theta_4 &= -\theta_6 \cdot v_4 \\
\kappa_5 - \theta_5 &= -\theta_6 \cdot v_5.
\end{aligned}$$

The variable input demand equation with and without uncertainty variables is used to quantify omitted variable bias. The variable input demand equation with uncertainty is given below:

$$\begin{aligned}
\left(x_2\right)_{it} = {} & \beta_0 + \beta_1 \cdot \frac{1}{\left(w_{21}\right)_t} + \beta_2 \cdot K_{it} + \beta_3 \cdot \frac{1}{c_t} + \beta_4 \cdot \ln y_{it} \cdot \frac{1}{c_t} + \beta_5 \cdot \ln\left(w_{21}\right)_t \cdot \frac{1}{c_t} \\
& + \beta_6 \cdot \ln c_t \cdot \frac{1}{c_t} + \beta_7 \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} + \beta_8 \cdot K_{it} \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} + \beta_9 \cdot \ln y_{it} \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} \\
& - \left[\beta_{10} + \beta_{15}\right] \cdot \left(K_{it} + I_{it}\right) \cdot \left(\sigma^2_{\ln w_2}\right)_t + \beta_{12} \cdot I_{it} - \beta_{13} \cdot I_{it} \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} \\
& + \left[-\beta_{11} + \beta_{14}\right] \cdot \left(K_{it} + I_{it}\right) \cdot \left(\sigma^2_{\ln c}\right)_t + \varepsilon_{\left(x_2\right)_{it}}.
\end{aligned} \quad (4.10)$$

The variable input demand without uncertainty variables is defined using equation (4.10). The model without uncertainty in the variable input demand equation is subdivided into two different models: one without the quasi-fixed factor price uncertainty—this is achieved by dropping the variance of quasi-fixed factor price in equation (4.10). Another one without the variable input price uncertainty—is achieved by dropping the variance of variable input price in equation (4.10). The auxiliary regression is used to quantify the bias on the estimates of coefficients.

Similarly, models with and without uncertainty are selected to quantify the omitted variable bias in the numeraire input demand equation. The numeraire variable input demand equation with uncertainty is given as:

$$
\begin{aligned}
(x_1)_{it} = {} & \eta_0 + \eta_1 \cdot K_{it} + \eta_2 \cdot \left(K^2\right)_{it} + \eta_3 \cdot \ln y_{it} \cdot K_{it} + \eta_4 \cdot \left(w_{21}\right)_t \cdot K_{it} + \eta_5 \cdot c_t \cdot K_{it} - \eta_6 \cdot K_{it} \cdot I_{it} \\
& + \eta_7 \cdot \ln y_{it} + \eta_8 \cdot \left(\ln y_{it}\right)^2 + \eta_9 \cdot \ln y_{it} \cdot \ln\left(w_{21}\right)_t + \eta_{10} \cdot \ln y_{it} \cdot \ln c_t - \eta_{11} \cdot \ln y_{it} \cdot I_{it} \\
& + \eta_{12} \cdot \ln\left(w_{21}\right)_t + \eta_{13} \cdot \left(\ln\left(w_{21}\right)_t\right)^2 + \eta_{14} \cdot \ln\left(w_{21}\right)_t \cdot \ln c_t - \eta_{15} \cdot \left(w_{21}\right)_t \cdot \left(x_2\right)_{it} \\
& - \eta_{16} \cdot \left(w_{21}\right)_t \cdot I_{it} + \eta_{17} \cdot \ln c_t + \eta_{18} \cdot \left(\ln c_t\right)^2 - \eta_{19} \cdot c_t \cdot I_{it} - \eta_{20} \cdot I_{it} \\
& - \left[\eta_{21} + \eta_{25}\right]\left(1 + K_{it} \cdot c_t\right) \cdot \left(\sigma^2_{\ln c}\right)_t - \left[\eta_{22} + \eta_{24}\right] \cdot \left(1 + K_{it} \cdot \left(w_{21}\right)_t\right) \cdot \left(\sigma^2_{\ln w_2}\right)_t \\
& - \eta_{23} \cdot \left(\sigma^2_{\ln y}\right)_t + \varepsilon_{(x_1)_{it}}.
\end{aligned}
\tag{4.11}
$$

Using the above equation, the model without uncertainty is defined in the numeraire input demand equation. Here, the numeraire demand without uncertainty consists of three different equations: without quasi-fixed factor price uncertainty; without variable input price uncertainty; and without output uncertainty. The respective auxiliary regression is employed to quantify the bias on the estimated model parameters.

4.2.2 Model specification for exploring impact of uncertainty

This subsection presents the model specification to analyze the impact of uncertainty on optimal factor allocations. In the net investment demand equation (4.4), uncertainty enters through an interaction term, $\left[\sigma^2_{(\ln c)_t} \cdot K_{it-1}\right]$. In this interaction term, K_{it-1} is considered as a moderator variable and the uncertainty variable is $\sigma^2_{\ln c}$. However, exploring the influence of quasi-fixed factor price uncertainty on the investment demand in the presence of the moderator variable, requires constitutive terms, such as $\sigma^2_{\ln c}$ and K_{it-1}. Therefore, the variance of the quasi-fixed factor price, $\sigma^2_{\ln c}$, is further considered as an explanatory variable, though this variable is not directly coming from the theoretical modeling approach in the investment demand equation (4.7). The resulting investment demand is given as:

$$
\begin{aligned}
I_{it} = {} & \theta_0 + \theta_1 \cdot \frac{1}{c_t} + \theta_2 \cdot \frac{1}{c_t} \cdot \ln y_{it} + \theta_3 \cdot \frac{1}{c} \cdot \ln\left(w_{21}\right)_t + \theta_4 \cdot \frac{1}{c_t} \cdot \ln c_t + \theta_5 \cdot K_{it-1} \\
& - \theta_6 \cdot \left(\sigma^2_{\ln c}\right)_t \cdot K_{it-1} + \theta_7 \cdot \left(\sigma^2_{\ln c}\right)_t.
\end{aligned}
\tag{4.12}
$$

Equation (4.12) helps to calculate meaningful marginal effects and standard errors. The conditional hypothesis of the net investment demand equation is as follows: investment is decreasing in the quasi-fixed factor price uncertainty for the different values of lagged quasi-fixed factor level. The marginal effect of $\sigma^2_{\ln c}$ on investment demand, I, is obtained by differentiating the equation (4.12) with respect to the variance of the quasi-fixed factor price, i.e., $\partial I / \partial \sigma^2_{\ln c} = -\theta_6 \cdot K_{it-1} + \theta_7$. The standard error of this term is calculated using the formula: $\sqrt{\left(K_{it-1}\right)^2 \cdot \operatorname{var}(\hat{\alpha}_6) + \operatorname{var}(\hat{\alpha}_7) + 2K_{it-1} \cdot \operatorname{cov}\left(\hat{\alpha}_6 \hat{\alpha}_7\right)}$. Note that the coefficients of the constitutive terms in the interaction models cannot be interpreted as average or unconditional marginal effects (cf. Brambor et al. 2006).

Even in the variable input demand equations, the output and input price uncertainty enters as interaction terms. The impact of uncertainty on the variable input demand equation (4.5) is evaluated using the same procedure. The additional constitutive terms, such as $\sigma^2_{\ln c}$ and $\sigma^2_{\ln w_2}$ are considered as explanatory variables in equation (4.5). The resulting variable input demand equation is given as:

$$
\begin{aligned}
\left(x_2\right)_{it} = {} & \beta_0 + \beta_1 \cdot \frac{1}{\left(w_{21}\right)_t} + \beta_2 \cdot K_{it} + \beta_3 \cdot \frac{1}{c_t} + \beta_4 \cdot \ln y_{it} \cdot \frac{1}{c_t} + \beta_5 \cdot \ln\left(w_{21}\right)_t \cdot \frac{1}{c_t} \\
& + \beta_6 \cdot \ln c_t \cdot \frac{1}{c_t} + \beta_7 \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} + \beta_8 \cdot K_{it} \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} + \beta_9 \cdot \ln y_{it} \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} \\
& - \beta_{10} \cdot K_{it} \cdot \left(\sigma^2_{\ln w_2}\right)_t - \beta_{11} \cdot K_{it} \cdot \left(\sigma^2_{\ln c}\right)_t + \beta_{12} \cdot I_{it} - \beta_{13} \cdot I_{it} \cdot \frac{1}{\left(w_{21}\right)_t} \cdot \frac{1}{c_t} \\
& + \beta_{14} \cdot I_{it} \cdot \left(\sigma^2_{\ln c}\right)_t - \beta_{15} \cdot I_{it} \cdot \left(\sigma^2_{\ln w_2}\right)_t + \beta_{16} \cdot \left(\sigma^2_{\ln c}\right)_t + \beta_{17} \cdot \left(\sigma^2_{\ln w_2}\right)_t .
\end{aligned}
\tag{4.13}
$$

The variable input demand consists of two different conditional hypotheses: first, the impact of the variance of the quasi-fixed factor price is negative on the variable input demand, given the positive values for investment and quasi-fixed factor levels. This is achieved by finding the simple slope (marginal effect) of the variance of the quasi-fixed factor price on the variable input demand. Differentiating equation (4.13) with respect to the variance of the quasi-fixed factor price $\sigma^2_{\ln c}$ yields marginal effect of $\sigma^2_{\ln c}$ on x_2, and is given as: $\partial x_2 / \partial \sigma^2_{\ln c} = -\beta_{11} \cdot K_{it} + \beta_{14} \cdot I_{it} + \beta_{16}$. The standard error of this term is calculated using the formula:

$$\mathrm{SE}\left(x_2\right)_{\sigma^2_{\ln c}} = \sqrt{\begin{array}{l}\left(K_{it}\right)^2 \cdot \mathrm{var}\left(\hat{\beta}_{11}\right)+\left(I_{it}\right)^2 \cdot \mathrm{var}\left(\hat{\beta}_{14}\right)+\mathrm{var}\left(\hat{\beta}_{16}\right)+2K_{it}\cdot \mathrm{cov}\left(\hat{\beta}_{11}\hat{\beta}_{16}\right)\\ +2I_{it}\cdot \mathrm{cov}\left(\hat{\beta}_{14}\hat{\beta}_{16}\right)+2K_{it}I_{it}\cdot \mathrm{cov}\left(\hat{\beta}_{11}\hat{\beta}_{14}\right)\end{array}}. \tag{4.14}$$

The second conditional hypothesis is that the variance of the variable input price has a negative effect on the variable input demand, given the positive values for the net investment and quasi-fixed factor stock. This is tested by calculating the marginal effect of variance of the variable input price on the variable input demand. Differentiating the equation (4.13) with respect to the variance of the variable input price $\sigma^2_{\ln w_2}$ results in: $\partial x_2 / \partial \sigma^2_{\ln w_2} = -\beta_{10} \cdot K_{it} - \beta_{15} \cdot I_{it} + \beta_{17}$. The standard error of the marginal effect of $\sigma^2_{\ln w_2}$ on the variable input demand, x_2, is calculated using the below formula:

$$\mathrm{SE}\left(x_2\right)_{\sigma^2_{\ln w_2}} = \sqrt{\begin{array}{l}\left(K_{it}\right)^2 \cdot \mathrm{var}\left(\hat{\beta}_{10}\right)+\left(I_{it}\right)^2 \cdot \mathrm{var}\left(\hat{\beta}_{15}\right)+\mathrm{var}\left(\hat{\beta}_{17}\right)+2K_{it}\cdot \mathrm{cov}\left(\hat{\beta}_{10}\hat{\beta}_{17}\right)\\ +2I_{it}\cdot \mathrm{cov}\left(\hat{\beta}_{15}\hat{\beta}_{17}\right)+2K_{it}I_{it}\cdot \mathrm{cov}\left(\hat{\beta}_{10}\hat{\beta}_{15}\right)\end{array}}. \tag{4.15}$$

Furthermore, the impact of different uncertainty variables on the numeraire input demand equation is evaluated using the same procedure (for details see Appendix B).

4.3 Simulated data

The proposed empirical factor demand equations are evaluated using simulated data. For this purpose, a panel dataset of 100 firms over ten years is constructed. Using known parameter values, data of 1000 sample observations are simulated in all the scenarios. Parameter assumptions for simulating the stochastic factor demand equations are presented in this subsection.

The discount rate (r) is assumed to be 5 per cent. The value function parameters are chosen based on previous empirical studies (Pietola and Myers 2000; Serra et al. 2010). Table 4.3 presents the values assigned for the value function parameters. The depreciation rate (δ) is assumed to be 5 per cent in all scenarios. A detailed procedure to generate variables in factor allocations is provided below.

Table 4.3: Known parameters values of the behavioral value function

Value function parameters	Known parameters values
Constant	$a_0 = 0.25$
First order parameters	$b_K = -0.25$, $b_y = 0.75$, $b_{w_{21}} = 0.5$ and $b_c = 0.5$
Second order parameters	$A_{KK} = 0.025$, $A_{yK} = 0.5$, $A_{yy} = 1$, $A_{w_2 y} = 0.75$, $A_{w_2 w_2} = -0.6$, $A_{cy} = 1.75$, $A_{cw_2} = 0.5$, $A_{cc} = -0.75$, $M_{cK} = 0.02$ and $A_{w_2 K} = 0.03$

Uncertainty variables

The uncertainty variables represent the conditional variances of input price variables $\left(\sigma^2_{(\ln w_1)_t}, \sigma^2_{(\ln w_2)_t}, \sigma^2_{(\ln c)_t}\right)$ over time. A time-varying volatility model is required to find out the effect of uncertainty on factor demand equations. Considering the independent univariate GARCH model further simplifies the process, therefore, co-variances were not considered in this simulations. Assume that each input price variable follows a univariate GARCH (1, 1) process. The conditional variance equation in the GARCH (1, 1) process is given as:

$$\sigma_t^2 = \omega_0 + \omega_1 \cdot a_{t-1}^2 + \xi_1 \cdot \sigma_{t-1}^2, \tag{4.16}$$

where σ_t^2 represents the conditional variance. The nonnegative conditional variance is ensured by assuming positive constraint for the coefficients of the GARCH model, i.e., $\omega_0 > 0$, $\omega_1 \geq 0$ and $\xi_1 \geq 0$. In the above equation, ω_0 represents the constant terms, ω_1 and ξ_1 denote ARCH and GARCH parameters, respectively. In addition to these conditions, the ARCH and GARCH parameters should add up to less than one $(\omega_1 + \xi_1 < 1)$, which satisfies stationary property (Tsay 2005). The unconditional variance is as follows: $\sigma^2 = \omega_0 / 1 - \omega_1 - \xi_1$, wherein the parameters ω_0, ω_1 and ξ_1 are the same as in the conditional variance equation (4.16).

The variable a_{t-1}^2 is calculated by using $a_t = \sigma_t \cdot e_t$, where a_t is referred to as shock or innovation of an asset return at time *t*, and e_t is a sequence of independent and identically distributed (iid) random variable with a mean of zero and unit variance. This term is also known as the innovation term, and is often assumed to be a standard normal, a standardized Student-*t* distribution, or a generalized error distribution. In this simulation, the innovation term is assumed to follows a standard normal distribution with a mean of zero and unit variance. A conditional standard deviation is represented by σ_t and is also defined as the positive square root of σ_t^2. Table 4.4 presents the known values of the GARCH parameters. Finally, conditional variance σ_t^2 in equation (4.16) is simulated using the known parameter values of the GARCH model, as well as a_{t-1}^2 and σ_{t-1}^2 variables.

Table 4.4: Know values of parameters in the conditional variance equation

Parameters	$\sigma^2_{(\ln w_1)_t}$	$\sigma^2_{(\ln w_2)_t}$	$\sigma^2_{(\ln c)_t}$
ω_0 (constant)	0.15	0.1	0.02
ω_1 (ARCH parameter)	0.45	0.6	0.54
ξ_1 (GARCH parameter)	0.4	0.3	0.4

Note: $\omega_1 + \xi_1 < 1$.

Input price variables

The input price variables (lnw_1, lnw_2, ln$c$) are assumed to follow an arithmetic Brownian motion:

$$\Delta z = \alpha \cdot \Delta t + \psi \cdot \Delta v, \tag{4.17}$$

where the state vector z is defined as $z = \left(\ln\left(w_1\right)_t, \ln\left(w_2\right)_t, \ln c_t\right)$, and α denotes a drift parameter, which was assumed to be zero in the theoretical modeling approach (section 3.2.2). The standard deviation, ψ, is calculated using the conditional variance, σ_t^2, i.e., a value from the univariate GARCH model. A standard Wiener process is defined as $\Delta v = \sqrt{\Delta t} \cdot \varepsilon_{gbm}$, wherein $\sqrt{\Delta t}$ denotes the length of time interval and ε_{gbm} is a normally distributed error with mean zero and variance one. Using the information from stochastic process, the logarithmic and absolute price variables are generated. The details of the derivation of log and absolute prices of variable and quasi-fixed factors are given in Table 4.5.

Table 4.5: Input price derivations using arithmetic Brownian motion

	Logarithmic prices	Absolute prices
Variable input price (w_1)	$\Delta z = \psi \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}$ $\Delta(\ln w_1) = \psi \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}$ $\ln(w_1)_t - \ln(w_1)_{t-1} = \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}$ $\ln(w_1)_t = \ln(w_1)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}$	$\ln(w_1)_t = \ln(w_1)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}$ $(w_1)_t = \exp\left(\ln(w_1)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}\right)$ $(w_1)_t = \mathrm{e}^{\left(\ln(w_1)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}\right)}$ $(w_1)_t = (w_1)_{t-1} \cdot \mathrm{e}^{\left(\psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_1}\right)}$
Variable input price (w_2)	$\Delta z = \psi \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}$ $\Delta(\ln w_2) = \psi \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}$ $\ln(w_2)_t - \ln(w_2)_{t-1} = \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}$ $\ln(w_2)_t = \ln(w_2)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}$	$\ln(w_2)_t = \ln(w_2)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}$ $(w_2)_t = \exp\left(\ln(w_2)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}\right)$ $(w_2)_t = \mathrm{e}^{\left(\ln(w_2)_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}\right)}$ $(w_2)_t = (w_2)_{t-1} \cdot \mathrm{e}^{\left(\psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_{w_2}\right)}$
Quasi-fixed factor price (c)	$\Delta z = \psi \cdot \sqrt{\Delta t} \cdot \varepsilon_c$ $\Delta(\ln c) = \psi \cdot \sqrt{\Delta t} \cdot \varepsilon_c$ $\ln c_t - \ln c_{t-1} = \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_c$ $\ln c_t = \ln c_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_c$	$\ln c_t = \ln c_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_c$ $c_t = \exp\left(\ln c_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_c\right)$ $c_t = \mathrm{e}^{\left(\ln c_{t-1} + \psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_c\right)}$ $c_t = c_{t-1} \cdot \mathrm{e}^{\left(\psi_t \cdot \sqrt{\Delta t} \cdot \varepsilon_c\right)}$

To ensure linear homogeneity of the cost function, factor prices are normalized by one of the variable factor prices. In this simulation, the variable input price w_1 is considered as a numeraire input. Therefore, other variable inputs in the simulation are redefined as: $w_2^b = (\lambda_2 w_2 / \lambda_1 w_1) = \lambda_{21} w_{21}$, where λ_{21} is the AI of the input pair $(x_2,\, x_1)$.

Quasi-fixed factor, output, and gross investment variables

The quasi-fixed factor (K), output (y) and gross investment (I) variables are generated in a recursive way, and the procedure consists of four steps which were provided below.

First, the initial value of quasi-fixed factor K_0 is set to 5. This is to avoid negative values in the subsequent steps.

Second, using the initial value of the quasi-fixed factor, the individual- and time-specific output variable is simulated by employing the Cobb-Douglas production function:

$$\ln y_{it} = \ln B + \rho \cdot \ln K_{i,t-1} + \varepsilon_{y_{it}}, \tag{4.18}$$

where y_{it} denotes the output and is a function of the lag of the quasi-fixed factor, $K_{i,t-1}$. In the production function, B and ρ represent constant and slope parameters and assigned a value of 1 and 0.5, respectively. The error term, $\varepsilon_{y_{it}}$, is generated using a normal distribution with a mean of zero and a standard deviation of 0.3. The output variable is constructed using the equation (4.18).

Third, θ parameters in the gross investment demand equation (4.7) are calculated using the known values of the value function (Table 4.3) and inefficiency parameters. The disturbance term, $\varepsilon_{I_{it}}$, in the net investment demand equation is assumed to follow a normal distribution with zero mean and standard deviation equal to 0.00005. The gross investment variable, I_{it}, is constructed by simulating the investment demand function given in equation (4.7).

Fourth, the subsequent values of the quasi-fixed factor, K_{it}, are generated using the capital accumulation constraint given in equation (3.4). Inserting the discrete measure of net investment in equation (3.4) assists in solving K_{it}. The subsequent values of the quasi-fixed factor, K_{it}, are calculated using the below formula:

$$K_{it} = \frac{I_{it} + K_{i,t-1}}{\left(1+\delta\right)}. \tag{4.19}$$

Similarly, the subsequent values of output, investment and quasi-fixed factor variables, are calculated by repeating the same procedure.

Output uncertainty

However, the sophisticated time varying volatility model [in equation (4.16)] is not considered for generating the output uncertainty, $\sigma^2_{(\ln y)_{it}}$. This variable is calculated using standard deviation of the output variable.

Variable input

The β parameters in the variable input demand $\left(x_2\right)$ in equation (4.10) are calculated using the known values of the inefficiency and value function parameters (Table 4.3). To generate the error term in the variable input demand equation, a normal distribution with a mean of zero and standard deviation of 0.0006 is used. The variable input demand equation (4.10) simulates the variable input $(x_2)_{it}$.

Numeraire variable input

The η parameters in the numeraire demand (x_1) in equation (4.11) are calculated using the known values of both inefficiency and value function parameters (Table 4.3). For the error term $\varepsilon_{(x_1)_{it}}$, assume a normal distribution with a mean of zero and a standard deviation of 0.008. Since uncertainty variables are generated by using a simple univariate GARCH model, co-variance is treated as zero in the numeraire input demand equation. The numeraire variable input, $(x_1)_{it}$, is simulated by using the numeraire input demand equation (4.11).

4.4 Simulation results

In this section, simulated data is used in each scenario to estimate stochastic factor demand equations in (4.7), (4.10) and (4.11). The major aim here is to assess and quantify the bias on the estimates of the model parameters if uncertainty is ignored in the optimal factor allocation. Models with and without uncertainty are estimated in each scenario, including the benchmark. To obtain the estimates and their respective standard errors, the total number of repetitions or replications is set to 1000. The least squares technique is used to estimate the factor demand equations in all scenarios. This section presents the simulation results.

4.4.1 Estimated bias due to omitted variable

4.4.1.1 Investment demand equation

Benchmark results—investment demand equation

In the case of investment demand, the models with and without uncertainty (cf. equations (4.7) and (4.8)) are estimated in the benchmark (perfect efficiency measure). Table 4.6 summarizes the mean of 1000 replications of the estimates and their standard errors. The simulation results show that the model with uncertainty in the investment demand equation provides correct and significant estimates, whereas the model without uncertainty in the investment equation yields incorrect estimates.

To test for omitted variables, the Ramsey RESET is used. The null hypothesis is that each model should have no omitted variables. For the model with uncertainty, the p-value is higher than the usual threshold of 0.05 (Prob>F=0.9276), so the null hypothesis is not rejected, thus indicating that the model is well specified. For the model without uncertainty, however, the p-value is lower than the threshold of 0.05 (Prob>F=0.009), which provides evidence of misspecification. The RESET results reveal the presence of omitted variable bias.

Table 4.6: Investment demand—benchmark results

Parameters	Known parameter values	With uncertainty			Without uncertainty		
		Estimate		SE	Estimate		SE
θ_0	2.5	2.5	***	2.7E-06	2.20	***	0.634
θ_1	0.0005	0.0005	***	3.0E-05	9.31	*	4.360
θ_2	0.00175	0.00175	***	9.4E-07	0.79	***	0.121
θ_3	0.0005	0.0005	***	2.4E-05	-8.18		6.139
θ_4	-0.00075	-0.00076	***	1.9E-05	-0.07		3.720
θ_5	0.08	0.08	***	5.8E-09	0.01	***	0.002
θ_6	-0.5	-0.5	***	4.6E-08	--		--
RESET result		F(3, 989) = 0.15 Prob > F = 0.9276			F(3, 990) = 3.88 Prob > F = 0.0090		

Note: Asterisks *** and * denote statistical significance at the 1% and 10% level, respectively; SE represents standard error.

The bias on the coefficients of interest was further quantified by running an auxiliary regression for the omitted variable (variance of quasi-fixed factor price) on the included variables in investment demand. The auxiliary regression result for the excluded variable (variance of quasi-fixed factor price) is given as:

$$\begin{aligned}\left[K_{it-1}\cdot\left(\sigma^2_{\ln c}\right)_t\right] &= 0.6-18.62\left(\frac{1}{c_t}\right)-1.58\left(\frac{1}{c_t}\cdot\ln y_{it}\right)+16.36\left(\frac{1}{c_t}\cdot\ln\left(w_{21}\right)_t\right)\\ &\quad+0.14\left(\frac{1}{c_t}\cdot\ln c_t\right)+0.14\left(K_{it-1}\right).\end{aligned} \tag{4.20}$$

Using this information, bias is calculated by multiplying the coefficient of the omitted variable by the coefficients in the auxiliary regression (see Table 4.7). The benchmark result in Table 4.7 indicates that the bias on the estimated coefficients θ_1 is increasing, whereas θ_3 is decreasing compared to the remaining estimates.

Table 4.7: Investment demand—bias calculation in the Benchmark

Parameters	Difference between the estimates of the models without and with uncertainty	Bias = (coefficient of the omitted variable) x (coefficients in the auxiliary regression)
θ_0	2.20 - 2.5 = -0.30	(-0.5) × 0.6 = -0.30
θ_1	9.31 - 0.0005 = 9.31	(-0.5) × (-18.62) = 9.31
θ_2	0.79 - 0.00175 = 0.79	(-0.5) × (-1.58) = 0.79
θ_3	-8.18 - 0.0005 = -8.18	(-0.5) × 16.36 = -8.18
θ_4	-0.07 - (-0.00076) = -0.07	(-0.5) × 0.14 = -0.07
θ_5	0.01 - 0.08 = -0.07	(-0.5) × 0.14 = -0.07

Scenario 1 results—investment demand equation

Table 4.8 presents the estimation result for scenario 1 (change in τ_K). The result implies that the model with uncertainty provides correct and significant estimates, whereas the result for the model without uncertainty shows that the estimates are incorrect and have the wrong signs. Here, one can see that the results of the model with uncertainty are better than those of the model without uncertainty.

Table 4.8: Investment demand—scenario 1 results

	Known parameters	With uncertainty			Without uncertainty			Bias
		Estimate		SE	Estimate		SE	
θ_0	2.5	2.5	***	2.8E-06	2.21	*	1.136	-1 × 0.29 = -0.29
θ_1	0.001	0.001	***	3.0E-05	16.93	*	8.200	-1 × -16.93 = 16.93
θ_2	0.0035	0.0035	***	9.6E-07	1.19	***	0.222	-1 × -1.18 = 1.18
θ_3	0.001	0.001	***	2.4E-05	-12.95		10.583	-1 × 12.96 = -12.96
θ_4	-0.0015	-0.0015	***	1.9E-05	-0.86		6.716	-1 × 0.86 = -0.86
θ_5	0.11	0.11	***	6.1E-09	-0.02	***	0.003	-1 × 0.13 = -0.13
θ_6	-1	-1	***	4.6E-08	--		--	--
RESET result		F(3, 989) = 0.17 Prob > F = 0.9174			F(3, 990) = 57.63 Prob > F = 0.0000			

Note: Asterisks *** and * denote statistical significance at the 1% and 10% level, respectively; SE represents standard error; Bias = (coefficient of the omitted variable) x (coefficients in the auxiliary regression).

For the model without uncertainty, the results of RESET show that there is evidence of misspecification, because the p-value is lower than the threshold of 0.05 (Prob > F = 0.0000) and fails to reject the null hypothesis for a model having no omitted variables; this indicates that variables have been omitted and reveals that ignoring uncertainty leads to biased estimates of the model parameters. The bias was further quantified by using the results of the auxiliary regression:

$$
\begin{aligned}
\left[K_{it-1}\cdot\left(\sigma_{\ln c}^{2}\right)_{t}\right] &= 0.29-16.93\left(\frac{1}{c_t}\right)-1.18\left(\frac{1}{c_t}\cdot\ln y_{it}\right)+12.96\left(\frac{1}{c_t}\cdot\ln\left(w_{21}\right)_t\right)\\
&\quad +0.86\left(\frac{1}{c_t}\cdot\ln c_t\right)+0.13\left(K_{it-1}\right).
\end{aligned}
\tag{4.21}
$$

In Table 4.8, the results show that there is an increasing bias on the estimates of model parameters compared to the benchmark case. Bias on the estimated coefficients θ_1 and θ_2 are considerably increasing compared to the benchmark. On the contrary, the estimated coefficient θ_3 is decreasing compared to the benchmark scenario. A change in the TI parameter of net investment (scenario 1) increases the bias on the estimated coefficients.

Scenario 2 results—investment demand equation

The results of scenario 2 (changes in $\sigma_{\ln c}^{2}$) are presented in Table 4.9. For the model with uncertainty, the RESET result rejects the null hypothesis of no omitted variables (Prob > F = 0.899), whereas for the model without uncertainty, the RESET result provides evidence of omitted variables (Prob > F = 0.0000). This signifies that the omission of the uncertainty variable results in biased estimates of the model parameters. The bias in the estimates was further quantified using the results of the auxiliary regression:

$$
\begin{aligned}
\left[K_{it-1}\cdot\left(\sigma_{\ln c}^{2}\right)_{t}\right] &= 0.17-35.13\left(\frac{1}{c_t}\right)-1.92\left(\frac{1}{c_t}\cdot\ln y_{it}\right)+23.84\left(\frac{1}{c_t}\cdot\ln\left(w_{21}\right)_t\right)\\
&\quad +5.14\left(\frac{1}{c_t}\cdot\ln c_t\right)+0.25\left(K_{it-1}\right).
\end{aligned}
\tag{4.22}
$$

Table 4.9. The model with uncertainty provides significant estimates with expected signs. Meanwhile, the result of the model without uncertainty indicates incorrect estimates and wrong signs. For the model with uncertainty, the RESET result rejects the null hypothesis of no omitted variables (Prob > F = 0.899), whereas for the model without uncertainty, the RESET result provides evidence of omitted variables (Prob > F = 0.0000). This signifies that the omission of the uncertainty variable results in biased estimates of the model parameters. The bias in the estimates was further quantified using the results of the auxiliary regression:

$$\left[K_{it-1}\cdot\left(\sigma^2_{\ln c}\right)_t\right]=0.17-35.13\left(\frac{1}{c_t}\right)-1.92\left(\frac{1}{c_t}\cdot\ln y_{it}\right)+23.84\left(\frac{1}{c_t}\cdot\ln\left(w_{21}\right)_t\right)$$
$$+5.14\left(\frac{1}{c_t}\cdot\ln c_t\right)+0.25\left(K_{it-1}\right). \tag{4.22}$$

Table 4.9: Investment demand—scenario 2 results

Parameters	Known parameter values	With uncertainty		Without uncertainty		Bias
		Estimate	SE	Estimate	SE	
θ_0	2.5	2.5 ***	2.8E-06	2.41 *	1.009	-0.5 × 0.17 = -0.09
θ_1	0.0005	0.0005 ***	3.0E-05	17.56 *	7.443	-0.5 × -35.13 = 17.56
θ_2	0.00175	0.00175 ***	9.7E-07	0.96 ***	0.200	-0.5 × -1.92 = 0.96
θ_3	0.0005	0.0005 ***	2.4E-05	-11.92	9.421	-0.5 × 23.84 = -11.92
θ_4	-0.00075	-0.00076 ***	1.9E-05	-2.57	6.010	-0.5 × 5.14 = -2.57
θ_5	0.08	0.08 ***	6.6E-09	-0.04 ***	0.002	-0.5 × 0.25 = -0.12
θ_6	-0.5	-0.5 ***	2.5E-08	--	--	--
RESET result		F(3, 989) = 0.20 Prob > F = 0.8990		F(3, 990) = 45.23 Prob > F = 0.0000		

Note: Asterisks *** and * denote statistical significance at the 1% and 10% level respectively; SE represents standard error; Bias = (coefficient of the omitted variable) x (coefficients in the auxiliary regression).

Here, the bias on the estimates of model parameters increases with increasing levels of the uncertainty variable (variance of quasi-fixed factor price). In For the model with uncertainty, the RESET result rejects the null hypothesis of no omitted variables (Prob > F = 0.899), whereas for the model without uncertainty, the RESET result provides evidence of omitted variables (Prob > F = 0.0000). This signifies that the omission of the uncertainty variable results in biased estimates of the model parameters. The bias in the estimates was further quantified using the results of the auxiliary regression:

$$\left[K_{it-1}\cdot\left(\sigma^2_{\ln c}\right)_t\right]=0.17-35.13\left(\frac{1}{c_t}\right)-1.92\left(\frac{1}{c_t}\cdot\ln y_{it}\right)+23.84\left(\frac{1}{c_t}\cdot\ln\left(w_{21}\right)_t\right)$$
$$+5.14\left(\frac{1}{c_t}\cdot\ln c_t\right)+0.25\left(K_{it-1}\right). \tag{4.22}$$

Table 4.9, the results of scenario 2 show that the bias on the estimated coefficient θ_1 is increasing compared to the benchmark estimate. In addition, the bias on the estimated coefficients θ_3 and θ_4 is decreasing compared to the benchmark estimates. This indicates that the bias is decreasing for θ_3 and θ_4 with an increasing uncertainty variable.

In addition to the scenarios presented above, a scenario of change in the AI parameter of the variable input price is considered in the investment demand equation. The result of this scenario is similar to the benchmark case, indicating that the increase in the AI parameter of the variable input price has no effect on investment demand. Hence, the result of this scenario is dropped in this simulation.

In the investment demand equation, identifying parameters using estimates is only possible for the model with uncertainty. That is, in the benchmark, as well as in scenarios 1 and 2, one can identify all the parameters using the estimated results, whereas in the scenario of change in the AI of the variable input price, parameters λ_{21} and b_c could not be identified. The identification problem is more prominent in the variable and numeraire input demand equations.

The adjustment rate, $\left(r - M_{cK} - 0.5\sigma^2_{\ln c}\right)$, is calculated for the model with uncertainty (for details see Appendix C). The adjustment rate depends on the discount rate as well as the value of the M_{cK} parameter. In all scenarios in the investment demand equation, the discount rate $r = 0.05$, value of $M_{cK} = 0.02$ and $\sigma^2_{\ln c}$ variable (except in scenario 2) are considered to be the same as the benchmark. Therefore, the adjustment rate is compared between the benchmark and scenario 2. In the benchmark scenario, the adjustment rate is -0.025, which means the quasi-fixed factor requires 40 years to the long-run equilibrium. In the case of scenario 2 (change in $\sigma^2_{\ln c}$), the adjustment rate is -0.08, implying that the quasi-fixed factor takes approximately 13 years to adjust to the long-run optimal level. This result indicates that increasing the uncertainty level speeds up the adjustment process. Further, to examine how the adjustment rate performs for the change in the value of M_{cK} to -0.08. In the benchmark scenario, the adjustment rate is 0.075, implying that a quasi-fixed factor takes approximately 13 years to adjust to the long-run optimal level. In scenario 2, the adjustment rate is 0.02, which means the quasi-fixed factor requires 50 years to adjust to the long-run equilibrium. This result indicates that increasing the uncertainty level reduces investment and slows down the adjustment process. The adjustment process in the previous empirical studies is presented in Table C.1 (Appendix C).

4.4.1.2 Variable input demand equation

Benchmark results—variable input demand equation

The simulated variable input demand was estimated using the least squares technique, and the result shows that the investment variable was omitted. This is due to the multicollinearity problem, because investment demand is considered one of the independent variables in simulating the variable input demand. To solve this problem, some of the independent variables were centered in the variable input demand equation. Using these centered variables; a variable input demand equation (4.10) was simulated again and estimated using the least squares technique. This holds true for the benchmark and other scenarios of variable input demand; their simulation results are provided below.

Table 4.10 depicts the benchmark (perfect efficiency measures) results for the variable input demand equation (4.10). In the model with uncertainty, some of the estimates are correct and significant. On the contrary, for the model without uncertainty of quasi-fixed and variable input prices, the results show that the estimates are incorrect but significant. The RESET result indicates the presence of omitted variables for the cases without quasi-fixed factor price uncertainty (Prob > F = 0.0000) and without variable input price uncertainty (Prob > F = 0.0000).

The bias was quantified using the results of auxiliary regression of the excluded variables on the included variables in the variable input demands (4.23) and (4.24). The auxiliary regression result for the excluded variable (variance of quasi-fixed factor price) on the included variables in the variable input demand is as follows:

$$\begin{aligned}\left(K_{it}+I_{it}\right)\cdot\left(\sigma_{\ln c}^{2}\right)_{t} ={}& -0.74-0.44\frac{1}{\left(w_{21}\right)_{t}}+0.28K_{it}+0.78\frac{1}{c_{t}}-0.59\ln y_{it}\frac{1}{c_{t}} \\ & -122.37\ln\left(w_{21}\right)_{t}\frac{1}{c_{t}}-10.47\ln c_{t}\frac{1}{c_{t}}-181.85\frac{1}{\left(w_{21}\right)_{t}}\frac{1}{c_{t}} \\ & -1.02K_{it}\frac{1}{\left(w_{21}\right)_{t}}\frac{1}{c_{t}}+4.67\ln y_{it}\frac{1}{\left(w_{21}\right)_{t}}\frac{1}{c_{t}}-0.11I_{it} \\ & +9.26I_{it}\frac{1}{\left(w_{21}\right)_{t}}\frac{1}{c_{t}}+0.14\left(K_{it}+I_{it}\right)\left(\sigma_{\ln w_{2}}^{2}\right)_{t}.\end{aligned} \tag{4.23}$$

The auxiliary regression result for the excluded variable (variance of variable input price) on the included variables in the variable input demand is:

$$
\begin{aligned}
\left(K_{it}+I_{it}\right)\cdot\left(\sigma^2_{\ln w_2}\right) = & -5.23+5.92\frac{1}{\left(w_{21}\right)_t}-0.11K_{it}-8.59\frac{1}{c_t}+3.57\cdot\ln y_{it}\frac{1}{c_t} \\
& +476.88\ln\left(w_{21}\right)_t\frac{1}{c_t}+42.54\ln c_t\frac{1}{c_t}+708.10\frac{1}{\left(w_{21}\right)_t}\frac{1}{c_t} \\
& +6.30K_{it}\frac{1}{\left(w_{21}\right)_t}\frac{1}{c_t}-33.90\ln y_{it}\frac{1}{\left(w_{21}\right)_t}\frac{1}{c_t}+0.13I_{it} \\
& -24.45I_{it}\frac{1}{\left(w_{21}\right)_t}\frac{1}{c_t}+6.48\left(K_{it}+I_{it}\right)\left(\sigma^2_{\ln c}\right)_t .
\end{aligned}
\tag{4.24}
$$

For the model without quasi-fixed factor price uncertainty, the bias on the estimated coefficients decreases, but the coefficients β_5 and β_7 are more biased, whereas for the model without variable input price uncertainty, the bias of the estimated coefficients increases, with the estimates of coefficient β_5 and β_7 being particularly more biased compared to the other estimates.

Scenario 1 results—variable input demand equation

Table 4.11 presents the results of scenario 1 (change in τ_K), which consist of an average of 1000 replications of the estimates and their standard errors. Some of the estimated coefficients are incorrect and not significant for both models, both with and without uncertainty. In addition, here the RESET results reject the null hypothesis, as the models do not have omitted variables. Estimated coefficients β_5 and β_7 are more biased compared to other estimates. The result for the model without variable input price uncertainty indicates a decrease in bias on the estimated coefficients compared to the benchmark estimates.

Scenario 2 results—variable input demand equation

The result of scenario 2 (change in $\sigma^2_{\ln c}$) is presented in Table 4.12. The estimated coefficients are incorrect and with wrong signs, but they are significant in the model without uncertainty. For the model without quasi-fixed factor price uncertainty, the bias on the estimated coefficients β_5 and β_7 is increasing compared to the other estimates. However, for the model without variable input price uncertainty, the estimated coefficients are less biased compared to the benchmark and scenario 1 results.

Table 4.10: Variable input demand—benchmark results

Parameters	Known parameter values	With uncertainty		Without uncertainty				Bias	
				Without varlnc		Without varlnw2		Without varlnc	Without varlnw2
		Estimate	SE	Estimate	SE	Estimate	SE		
β_0	2	2***	0.004	1.784***	0.398	3.562*	1.009	-0.21	1.64
β_1	-0.005	-0.006***	0.000	-0.078	0.149	-1.359	1.672	-0.12	-1.86
β_2	0.0024	0.0024***	0.000	0.010***	0.000	0.036***	0.002	0.01	0.03
β_3	0.000015	5.02E-05	0.001	-0.247	0.516	-1.938	4.879	0.22	2.71
β_4	0.0000525	6.79E-05	0.000	-0.170***	0.026	-1.123***	0.205	-0.17	-1.12
β_5	0.000015	-0.0199	0.022	-34.806	19.096	-149.608	155.097	-34.87	-150.22
β_6	-0.0000225	0.0010	0.003	-3.137**	1.082	-14.919	8.005	-2.99	-13.40
β_7	-0.000125	-0.0252	0.030	-51.800*	26.772	-222.849	215.749	-51.83	-223.05
β_8	0.0000125	0.000097	0.000	-0.289***	0.053	-1.982***	0.453	-0.29	-1.98
β_9	0.00025	0.00017	0.001	1.329***	0.230	10.668***	1.879	1.33	10.68
β_{12}	-0.018	-0.018***	0.000	-0.049***	0.002	-0.059***	0.015	-0.03	-0.04
β_{13}	-0.00025	-0.00175	0.001	2.635**	0.936	7.694	7.586	2.64	7.70
β_{1015}	-0.315	-0.315***	0.000	-0.275***	0.000	--	--	0.04	--
β_{1114}	0.285	0.285***	0.000	--	--	-1.757***	0.017	--	-2.04
RESET result		F(3, 982) = 1.79 Prob > F = 0.1465		F(3, 983) = 31.23 Prob > F = 0.0000		F(3, 983) = 26.38 Prob > F = 0.0000			

Note: Asterisks *** and * denote statistical significance at the 1% and 10% level, respectively; SE represents standard error; varlnc is the variance of quasi-fixed input price; varlnw2 is the variance of variable input price.

Table 4.11: Variable input demand—scenario 1 results

Parameters	Known parameter values	With uncertainty			Without uncertainty						Bias	
					Without varlnc			Without varlnw2			Without varlnc	Without varlnw2
		Estimate		SE	Estimate		SE	Estimate		SE		
β_0	2	2	***	0.003	1.894	*	0.747	1.319		2.295	-0.11	-0.68
β_1	0.02	0.02	***	0.001	0.236		1.632	1.330		8.595	0.21	1.27
β_2	0.0033	0.0033	***	0.000	0.015	***	0.001	0.025	***	0.002	0.01	0.02
β_3	0.000045	-0.0005		0.001	0.014		0.741	0.476		3.166	0.08	0.79
β_4	0.000105	0.000115	**	0.000	-0.283	***	0.049	-0.850	***	0.168	-0.28	-0.85
β_5	0.00003	-0.0309		0.024	-31.413		43.046	-31.855		196.830	-31.53	-32.38
β_6	-4.5E-05	0.0016		0.003	-5.565	**	2.123	-7.606		6.968	-5.51	-7.29
β_7	-0.00025	-0.0481		0.033	-40.250		59.950	-48.701		272.386	-40.29	-48.80
β_8	0.000025	-0.0002		0.000	-0.476	***	0.079	-1.748	***	0.335	-0.48	-1.75
β_9	0.0005	0.0004		0.001	1.929	***	0.408	6.833	***	1.585	1.93	6.85
β_{12}	-0.036	-0.036	***	0.000	-0.058	***	0.003	-0.040	***	0.011	-0.02	0.00
β_{13}	-0.0005	-0.0008		0.001	2.247	*	0.920	1.640		3.647	2.25	1.65
β_{1015}	-0.315	-0.315	***	0.000	-0.235	***	0.001	--		--	0.08	--
β_{1114}	0.57	0.57	***	0.001	--		--	-1.464	***	0.017	--	-2.03
RESET result		F(3, 982) = 0.85 Prob > F = 0.4667			F(3, 983) = 100.29 Prob > F = 0.0000			F(3, 983) = 14.88 Prob > F = 0.0000				

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error; varlnc is the variance of quasi-fixed factor price; varlnw2 is the variance of variable input price.

Table 4.12: Variable input demand—scenario 2 results

Parameters	Known parameter values	With uncertainty			Without uncertainty						Bias	
					Without varlnc			Without varlnw2			Without varlnc	Without varlnw2
		Estimate		SE	Estimate		SE	Estimate		SE		
β_0	2	2.004	***	0.003	1.784	*	0.711	1.101		2.135	-0.22	-0.91
β_1	-0.005	-0.005	***	0.000	0.231		1.427	1.226		7.402	0.24	1.22
β_2	0.0024	0.002	***	0.001	0.012	***	0.001	0.023	***	0.003	0.01	0.02
β_3	0.000015	-0.00047		0.001	-0.095		0.654	-0.074		2.797	-0.06	0.10
β_4	0.0000525	0.00006		0.000	-0.246	***	0.042	-0.736	***	0.138	-0.25	-0.74
β_5	0.000015	-0.031		0.025	-21.824		38.089	2.876		181.821	-22.49	2.49
β_6	-0.0000225	0.0095		0.009	-4.215	*	1.826	-4.547		5.807	-4.27	-4.30
β_7	-0.000125	-0.045		0.034	-28.116		53.357	-7.801		252.482	-28.86	-7.82
β_8	0.0000125	0.004		0.004	-0.424	***	0.073	-1.732	***	0.308	-0.43	-1.74
β_9	0.00025	0.00015		0.001	1.237	***	0.339	4.215	**	1.282	1.27	4.25
β_{12}	-0.018	-0.018	***	0.000	-0.033	***	0.003	-0.009		0.010	-0.02	0.01
β_{13}	-0.00025	-0.0013		0.001	1.959	*	0.864	1.180		3.413	2.02	1.20
β_{1015}	-0.315	-0.316	***	0.001	-0.235	***	0.001	--		--	0.08	--
β_{1114}	0.285	0.292	***	0.007	--		--	-0.736	***	0.009	--	0.94
RESET result		$F(3, 982) = 0.79$ Prob > F = 0. 5011			$F(3, 983) = 159.33$ Prob > F = 0.0000			$F(3, 983) = 51.05$ Prob > F = 0.0000				

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error; varlnc is the variance of quasi-fixed factor price; varlnw2 is the variance of variable input price.

Scenario 3 results—variable input demand equation

Table 4.13 depicts the results of scenario 3 (change in $\sigma^2_{\ln w_2}$); the estimated coefficients are similar to the benchmark results for the model with uncertainty and the model without quasi-fixed factor price uncertainty. However, the estimated coefficients are incorrect and have the wrong signs in the model without variable input price uncertainty. Also, here the RESET results reject the null hypothesis of a model having no omitted variables. The bias in the coefficients of the model without quasi-fixed factor price uncertainty is similar to the benchmark results. In the case of the model without variable input price uncertainty, the bias on the coefficients is more pronounced compared to other scenarios. In particular, the bias in coefficients β_5 and β_7 is more evident compared to the other coefficients. The results of scenario 3 indicate that the bias in the estimates of the model without variable input price uncertainty is increasing compared to the results of other scenarios.

Table 4.13: Variable input demand—scenario 3 results

Parameters	Known parameter values	With uncertainty			Without uncertainty						Bias	
					Without varlnc			Without varlnw2			Without varlnc	Without varlnw2
		Estimate		SE	Estimate		SE	Estimate		SE		
β_0	2	2.004	***	0.004	1.784	***	0.398	5.120	*	2.015	-0.21	3.28
β_1	-0.005	-0.006	***	0.000	-0.078		0.149	-2.712		3.344	-0.12	-3.73
β_2	0.0024	0.0024	***	0.000	0.010	***	0.000	0.069	***	0.005	0.01	0.07
β_3	0.000015	0.0000		0.001	-0.247		0.516	-3.876		9.759	0.22	5.41
β_4	0.0000525	0.0001		0.000	-0.170	***	0.026	-2.246	***	0.410	-0.17	-2.25
β_5	0.000015	-0.02		0.022	-34.806		19.096	-299.200		310.199	-34.87	-300.43
β_6	-0.0000225	0.0010		0.003	-3.137	**	1.082	-29.838		16.010	-2.99	-26.80
β_7	-0.000125	-0.025		0.030	-51.800	*	26.772	-445.678		431.504	-51.83	-446.10
β_8	0.0000125	0.00010		0.000	-0.289	***	0.053	-3.964	***	0.905	-0.29	-3.97
β_9	0.00025	0.00017		0.001	1.329	***	0.230	21.336	***	3.758	1.33	21.36
β_{12}	-0.018	-0.018	***	0.000	-0.049	***	0.002	-0.100	**	0.030	-0.03	-0.08
β_{13}	-0.00025	-0.00175		0.001	2.635	*	0.936	15.389		15.173	2.64	15.41
β_{1015}	-0.315	-0.315	***	0.000	-0.295		0.000	--		--	0.02	--
β_{1114}	0.285	0.285	***	0.000	--		--	-3.799	***	0.035	--	-4.08
RESET result		F(3, 982) = 1.86 Prob > F = 0.1353			F(3, 983) = 32.27 Prob > F = 0.0000			F(3, 983) = 27.25 Prob > F = 0.0000				

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error; varlnc is the variance of quasi-fixed factor price; varlnw2 is the variance of variable input price.

4.4.1.3 Numeraire input demand equation

Benchmark results—numeraire input demand equation

Investment demand and variable input demand enter as independent variables in the numeraire input demand equation (4.11). To overcome the collinearity problem, these independent variables were centered and used to simulate the numeraire input demand, and then to estimate the simulated numeraire demand using the least squares technique. This procedure is the same for all scenarios, including the benchmark case.

Table 4.14 presents the benchmark (perfect efficiency measure) result of the numeraire input demand. The result of the model with uncertainty shows that most of the estimates are correct and significant, whereas the results of the models without uncertainty indicate that the estimates are incorrect, with the wrong signs, and not significant. The RESET results reveal evidence of omitted variables in the model without quasi-fixed factor price uncertainty (Prob > F = 0.0000) and the model without variable input price uncertainty (Prob > F = 0.0000), but fails to reject the null hypothesis of no omitted variables in the case of the model without output uncertainty (Prob > F = 0.6658). Bias in the estimates of model parameters was further explored using results of auxiliary regression for the excluded variables on the included variables.

The result of auxiliary regression of the excluded variable (variance of quasi-fixed factor price) is as follows:

$$\begin{aligned}
\left(1+K_{it}c_t\right)\cdot\left(\sigma^2_{\ln c}\right)_t = {} & 139.95-1.09K_{it}-0.001\left(K^2\right)_{it}-0.002\ln y_{it}\cdot K_{it} \\
& +4.62\left(w_{21}\right)_t K_{it}+0.13c_tK_{it}+0.01K_{it}I_{it}+0.27\ln y_{it} \\
& -0.09\left(\ln y_{it}\right)^2-2.29\ln y_{it}\ln\left(w_{21}\right)_t-0.05\ln y_{it}\ln c_t \\
& -0.04\ln y_{it}I_{it}+32.97\ln\left(w_{21}\right)_t+129.36\left(\ln\left(w_{21}\right)_t\right)^2 \\
& -371.43\ln\left(w_{21}\right)_t\ln c_t+2.75\left(w_{21}\right)_t\left(x_2\right)_{it} \\
& -46.41\left(w_{21}\right)_t\cdot I_{it}-179.26\ln c_t+38.85\left(\ln c_t\right)^2-0.94c_tI_{it} \\
& -1730.67I_{it}-0.29\left(1+K_{it}\left(w_{21}\right)_t\right)\left(\sigma^2_{\ln w_2}\right)_t+0.31\left(\sigma^2_{\ln y}\right)_t .
\end{aligned} \tag{4.25}$$

The result of auxiliary regression of the excluded variable (variance of variable input price) is as follows:

$$\begin{aligned}\left(1+K_{it}\left(w_{21}\right)_t\right)\cdot\left(\sigma^2_{\ln w_2}\right)_t &= -8.21+1.23K_{it}+0.0001\left(K^2\right)_{it}-0.005\ln y_{it}K_{it} \\ &-0.09\left(w_{21}\right)_t K_{it}-0.01c_tK_{it}+0.002K_{it}I_{it}-0.13\ln y_{it} \\ &+0.06\left(\ln y_{it}\right)^2+0.26\ln y_{it}\ln\left(w_{21}\right)_t-0.24\ln y_{it}\ln c_t \\ &+0.01\ln y_{it}I_{it}-0.55\ln\left(w_{21}\right)_t-11.02\left(\ln\left(w_{21}\right)_t\right)^2 \\ &+60.70\ln\left(w_{21}\right)_t\ln c_t-3.33\left(w_{21}\right)_t\left(x_2\right)_{it} \\ &+0.34\left(w_{21}\right)_t I_{it}+12.67\ln c_t-6.86\left(\ln c_t\right)^2+0.04c_tI_{it} \\ &+87.41I_{it}+0.02\left(1+K_{it}c_t\right)\left(\sigma^2_{\ln c}\right)_t+0.07\left(\sigma^2_{\ln y}\right)_t.\end{aligned} \tag{4.26}$$

The result of auxiliary regression of the excluded variable (variance of output level) is as follows:

$$\begin{aligned}\left(\sigma^2_{\ln y}\right)_t &= 0.46+0.02K_{it}+0.0001\left(K^2\right)_{it}-0.003\ln y_{it}K_{it}-0.05\left(w_{21}\right)_t\cdot K_{it} \\ &+0.01c_tK_{it}-0.0002K_{it}I_{it}-0.34\ln y_{it}+0.10\left(\ln y_{it}\right)^2 \\ &+0.01\ln y_{it}\ln\left(w_{21}\right)_t+0.002\ln y_{it}\ln c_t+0.002\ln y_{it}I_{it}+0.2\ln\left(w_{21}\right)_t \\ &-1.13\left(\ln\left(w_{21}\right)_t\right)^2-1.41\ln\left(w_{21}\right)_t\ln c_t-0.01\left(w_{21}\right)_t\left(x_2\right)_{it} \\ &-0.05\left(w_{21}\right)_t I_{it}-0.03\ln c_t-0.14\left(\ln c_t\right)^2+0.02c_tI_{it}+0.004I_{it} \\ &-0.003\left(1+K_{it}c_t\right)\left(\sigma^2_{\ln c}\right)_t+0.003\left(1+K_{it}\left(w_{21}\right)_t\right)\left(\sigma^2_{\ln w_2}\right)_t.\end{aligned} \tag{4.27}$$

In the case of the model without quasi-fixed factor price uncertainty, the bias on the estimates is increasing compared to the other models without uncertainty (in Table 4.18). In particular, the bias on the estimated coefficients η_{12} and η_{20} is more evident compared to the remaining estimates. The bias on the estimated coefficients is relatively low for the model without output price uncertainty compared to the other models without uncertainty.

Scenario 1 results—numeraire input demand equation

Table 4.15 presents the estimated result of scenario 1 (change in τ_K) for numeraire input demand. Most of the estimates are correct and significant for the model with uncertainty. The RESET result is very weak for the model without output uncertainty (Prob > F = 0.2225) because it signifies that there is no evidence for the presence of omitted variables. However, for the models without quasi-fixed factor price uncertainty (Prob > F = 0.0009) and variable input price uncertainty (Prob > F = 0.0000), results indicate that there is evidence for the presence of omitted variables.

Table 4.14: Numeraire input demand—benchmark results

Parameters	Known parameter values	With uncertainty		Without uncertainty					
				Without varlnc		Without varlnw2		Without varlny	
		Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
eta0	500.0125	500.0126***	0.002	551.04***	45.758	499.31***	0.782	499.78***	0.031
eta1	-0.0125	-0.0126***	0.000	-0.41	1.035	0.34	0.360	-0.02**	0.007
eta2	0.000625	0.000625***	0.000	0.0004**	0.000	0.001***	0.000	0.001***	0.000
eta3	0.025	0.025***	0.000	0.02***	0.002	0.02***	0.001	0.03***	0.000
eta4	0.0015	0.0016***	0.000	1.69	1.464	0.03	0.016	0.03	0.022
eta5	-0.999	-0.999***	0.000	-0.95***	0.009	-1.00***	0.002	-1.00***	0.003
eta6	-0.025	-0.025***	0.000	-0.02***	0.004	-0.02***	0.001	-0.02***	0.000
eta7	0.0375	0.0369***	0.000	0.13	0.092	0.001	0.037	0.21***	0.001
eta8	0.025	0.025***	0.000	-0.01	0.036	0.04*	0.016	-0.02***	0.000
eta9	0.0375	0.0370***	0.001	-0.80*	0.314	0.11	0.091	0.03***	0.005
eta10	0.0875	0.0875***	0.001	0.07	0.182	0.02	0.058	0.09***	0.003
eta11	-0.5	-0.5***	0.000	-0.51***	0.007	-0.50***	0.003	-0.50***	0.000
eta12	0.025	0.0322***	0.007	12.04	12.988	-0.13	1.986	-0.07	0.120
eta13	-0.015	-0.1595	0.112	47.03	41.858	-2.59	9.640	0.40	0.794
eta14	0.025	-0.1148	0.119	-135.56	122.046	12.83	9.618	0.58	0.821
eta15	-1	-1***	0.001	0.004	0.592	-1.95***	0.009	-0.99***	0.007
eta16	-0.03	-0.03***	0.003	-16.95	14.186	-0.44***	0.082	0.001	0.043
eta17	0.025	0.022***	0.005	-65.34	62.259	1.41*	0.726	0.04*	0.019
eta18	-0.01875	-0.0524	0.032	14.12	17.770	-1.40	2.397	0.02	0.206
eta19	-0.02	-0.02***	0.000	-0.36***	0.052	-0.01	0.008	-0.03***	0.006
eta20	0.25	0.25***	0.000	-630.81	624.518	2.75	2.583	0.25***	0.002
eta2125	0.365	0.365***	0.000	--	--	0.37***	0.003	0.37***	0.002
eta2224	0.285	0.285***	0.000	0.18	0.132	--	--	0.28***	0.001
eta23	-0.5	-0.5***	0.000	-0.45***	0.049	-0.48***	0.017	--	--

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error.

Table 4.15: Numeraire input demand—scenario 1 results

Parameters	Known parameter values	With uncertainty		Without uncertainty					
				Without varlnc		Without varlnw2		Without varlny	
		Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
eta0	500.01	500.0126 ***	0.002	483.136 ***	20.497	502.908 ***	3.325	499.812 ***	0.006
eta1	-0.0125	-0.0127 ***	0.000	-6.423	6.810	1.103	1.103	-0.014 ***	0.002
eta2	0.000625	0.000625 ***	0.000	0.001 ***	0.000	0.001 ***	0.000	0.001 ***	0.000
eta3	0.025	0.025 ***	0.000	0.023 ***	0.001	0.025 ***	0.000	0.026 ***	0.000
eta4	0.0015	0.0020 ***	0.001	0.379	0.483	0.077 *	0.033	0.008	0.006
eta5	-0.999	-0.999 ***	0.000	-0.951 ***	0.005	-0.996 ***	0.003	-1.000 ***	0.002
eta6	-0.0125	-0.0125 ***	0.000	-0.008 ***	0.001	-0.012 ***	0.000	-0.012 ***	0.000
eta7	0.0375	0.0368 ***	0.000	0.071 **	0.027	0.034 ***	0.002	0.207 ***	0.001
eta8	0.025	0.025 ***	0.000	0.019	0.011	0.026 ***	0.001	-0.024 ***	0.000
eta9	0.0375	0.0371 ***	0.001	-0.400 **	0.120	0.019 *	0.009	0.032 ***	0.005
eta10	0.0875	0.0874 ***	0.001	-0.105 **	0.039	0.080 ***	0.005	0.087 ***	0.003
eta11	-0.25	-0.25 ***	0.000	-0.255 ***	0.002	-0.249 ***	0.000	-0.251 ***	0.000
eta12	0.025	0.0204 *	0.010	11.227	11.878	3.901	2.278	0.073	0.057
eta13	-0.015	-0.0879	0.125	-6.148	52.645	-0.711	9.576	0.432	0.833
eta14	0.025	-0.018	0.130	25.162	55.506	17.192	9.510	0.401	0.730
eta15	-1	-1 ***	0.002	-1.077 *	0.482	-2.066 ***	0.013	-0.997 ***	0.005
eta16	-0.015	-0.012 ***	0.001	-6.856	4.883	-0.335 ***	0.060	-0.019 *	0.008
eta17	0.025	0.018 **	0.005	-3.086	15.904	3.354	2.303	0.061 *	0.030
eta18	-0.01875	-0.03209	0.035	-8.642	12.582	-1.574	2.330	0.118	0.183
eta19	-0.01	-0.01 ***	0.001	-0.147 ***	0.021	-0.027 ***	0.007	-0.010 ***	0.002
eta20	0.125	0.125 ***	0.000	-197.615	197.683	0.099 ***	0.016	0.125 ***	0.002
eta2125	0.365	0.364 ***	0.001	-- --	--	0.344 ***	0.004	0.365 ***	0.002
eta2224	0.285	0.286 ***	0.001	-0.115	0.088	-- --	--	0.286 ***	0.002
eta23	-0.5	-0.5 ***	0.000	-0.537 ***	0.047	-0.498 ***	0.001		--

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error.

The bias of the estimates is quantified using auxiliary regression results. In the case of the model without quasi-fixed factor price uncertainty, the bias of the estimated coefficients is decreasing compared to the benchmark results (Table 4.18). In the model without quasi-fixed factor price uncertainty, the bias on the estimated coefficients η_0 and η_1 is particularly increasing compared to the estimated coefficients in the benchmark scenario. However, in the other models without uncertainty, the bias of the estimates is much less.

Scenario 2 results—numeraire input demand equation

Table 4.16 depicts the estimated result of scenario 2 (change in $\sigma^2_{\ln c}$) for the numeraire variable input demand. Most of the estimates are correct and significant for the model with uncertainty. The RESET indicates the wrong signal for all the models without uncertainty, except for the model without variable input price uncertainty (Prob > F = 0.0000); this implies the presence of omitted variables for this model. The bias in the estimates of the model without quasi-fixed factor price uncertainty is decreasing compared to the benchmark results, but increasing compared to the results from scenario 1 (see Table 4.18). In the model without variable input price uncertainty, bias on the estimated coefficients η_0 and η_1 is increasing compared to the benchmark and scenario 1 coefficients.

Scenario 3 results—numeraire input demand equation

In Table 4.17, the RESET results of scenario 3 (change in $\sigma^2_{\ln w_2}$) are similar to the benchmark results, in that evidence exists of the presence of omitted variables in the models without variance of quasi-fixed factor price (Prob > F = 0.0000) and variance of variable input price (Prob > F = 0.0000). For the model without output uncertainty, the RESET result signifies that there is no omitted variable (Prob > F = 0.7290). Meanwhile, the bias on the estimated coefficients is slightly increasing compared to the benchmark estimates without a quasi-fixed factor price uncertainty model. However, in the model without quasi-fixed factor price uncertainty, the bias on the estimated coefficients η_{12} and η_{20} is increasing compared to the remaining estimates (see Table 4.18).

Table 4.16: Numeraire input demand—scenario 2 results

Parameters	Known parameter values	With uncertainty			Without uncertainty								
					Without varlnc			Without varlnw2			Without varlny		
		Estimate		SE	Estimate		SE	Estimate		SE	Estimate		SE
eta0	500.0125	500.013	***	0.001	471.68	***	38.497	502.77	***	3.124	499.80	***	0.017
eta1	-0.0125	-0.0126	***	0.000	-12.88		13.727	1.12		1.121	-0.02	***	0.002
eta2	0.000625	0.00063	***	0.000	0.0009	***	0.000	0.0006	***	0.000	0.0006	***	0.000
eta3	0.025	0.025	***	0.000	0.02	***	0.002	0.02	***	0.000	0.03	***	0.000
eta4	0.0015	0.0013	***	0.000	0.28		0.640	0.07	*	0.033	0.01		0.011
eta5	-0.999	-0.999	***	0.000	-0.91	***	0.009	-1.00	***	0.002	-1.00	***	0.002
eta6	-0.025	-0.025	***	0.000	-0.02	***	0.003	-0.02	***	0.000	-0.02	***	0.000
eta7	0.0375	0.0368	***	0.000	0.08		0.054	0.03	***	0.002	0.20	***	0.001
eta8	0.025	0.025	***	0.000	0.02		0.021	0.03	***	0.001	-0.02	***	0.000
eta9	0.0375	0.037	***	0.001	-0.65	*	0.266	0.02	*	0.010	0.03	***	0.004
eta10	0.0875	0.0874	***	0.001	-0.27	**	0.090	0.08	***	0.005	0.09	***	0.002
eta11	-0.5	-0.5	***	0.000	-0.51	***	0.004	-0.50	***	0.000	-0.50	***	0.000
eta12	0.025	0.018	*	0.009	16.93		23.353	4.07		2.124	0.09		0.055
eta13	-0.015	-0.141		0.123	-19.65		97.852	-0.29		8.952	0.63		0.536
eta14	0.025	-0.109		0.128	43.40		104.467	16.03		8.887	0.27		0.539
eta15	-1	-1	***	0.002	-0.51		0.942	-2.09	***	0.014	-1.00	***	0.005
eta16	-0.03	-0.03	***	0.002	-12.81		9.159	-0.35	***	0.060	-0.03	**	0.009
eta17	0.025	0.024	***	0.004	-4.19		31.599	2.72		2.143	0.03		0.024
eta18	-0.01875	-0.049		0.035	-16.48		22.994	-1.54		2.176	0.07		0.135
eta19	-0.02	-0.02	***	0.001	-0.28	***	0.038	-0.03	***	0.006	-0.02	***	0.002
eta20	0.25	0.25	***	0.001	-372.44		372.532	0.22	***	0.016	0.25	***	0.002
eta2125	0.365	0.365	***	0.001	--		--	0.36	***	0.002	0.37	***	0.002
eta2224	0.285	0.285	***	0.001	-0.36	*	0.165	--		--	0.28	***	0.002
eta23	-0.5	-0.5	***	0.000	-0.62	***	0.111	-0.50	***	0.001	--		--

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error.

Table 4.17: Numeraire input demand—scenario 3 results

Parameters	Known parameter values	With uncertainty		Without uncertainty					
				Without varlnc		Without varlnw2		Without varlny	
		Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
eta 0	500.0125	500.01 ***	0.003	507.24 ***	6.501	500.14 ***	0.304	499.67 ***	0.157
eta1	-0.0125	-0.01 ***	0.000	0.60	0.544	0.10	0.100	-0.02 *	0.009
eta2	0.000625	0.0006 ***	0.000	0.0005 ***	0.000	0.0007 ***	0.000	0.0006 ***	0.000
eta3	0.025	0.02 ***	0.000	0.02 ***	0.001	0.02 ***	0.002	0.03 ***	0.000
eta4	0.0015	0.002 ***	0.000	1.53	1.264	0.03	0.017	0.03	0.026
eta5	-0.999	-1.00 ***	0.000	-0.95 ***	0.008	-1.00 ***	0.004	-1.00 ***	0.004
eta6	-0.025	-0.03 ***	0.000	-0.02 ***	0.003	-0.03 ***	0.001	-0.02 ***	0.000
eta7	0.0375	0.04 ***	0.000	0.09	0.073	0.01	0.032	0.21 ***	0.001
eta8	0.025	0.03 ***	0.000	0.02	0.026	0.04 *	0.019	-0.02 ***	0.000
eta9	0.0375	0.04 ***	0.001	-0.78 **	0.240	0.03	0.018	0.03 ***	0.005
eta10	0.0875	0.09 ***	0.001	0.05	0.123	0.07 ***	0.011	0.09 ***	0.003
eta11	-0.5	-0.50 ***	0.000	-0.51 ***	0.005	-0.49 ***	0.005	-0.50 ***	0.000
eta12	0.025	0.01	0.017	50.23	28.207	1.31	2.332	0.14 *	0.059
eta13	-0.015	-0.09	0.126	11.01	57.582	-1.80	10.359	0.72	0.795
eta14	0.025	-0.001	0.132	3.73	41.137	14.16	10.265	0.62	0.819
eta15	-1	-1.00 ***	0.001	-0.52	0.271	-1.48 ***	0.004	-1.00 ***	0.004
eta16	-0.03	-0.03 ***	0.003	-15.14	12.448	-0.07	0.074	0.01	0.053
eta17	0.025	0.02 *	0.009	-12.19	15.693	-0.66	1.484	0.03	0.028
eta18	-0.01875	-0.03	0.037	-10.54	12.156	-1.32	2.530	0.02	0.206
eta19	-0.02	-0.02 ***	0.000	-0.38 ***	0.048	-0.04	0.021	-0.03 ***	0.004
eta20	0.25	0.25 ***	0.000	-555.47	547.08	1.51	1.347	0.25 ***	0.002
eta2125	0.365	0.37 ***	0.000	--	--	0.37 ***	0.003	0.36 ***	0.003
eta2224	0.285	0.14 ***	0.000	0.08	0.064	--	--	0.14 ***	0.001
eta23	-0.5	-0.50 ***	0.000	-0.45 ***	0.046	-0.47 ***	0.026	--	--

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error.

Table 4.18: Numeraire input demand—bias calculations in all scenarios

Parameters	Benchmark			Scenario 1			Scenario 2			Scenario 3		
	Without varlnc	Without varlnw2	Without varlny	Without varlnc	Without varlnw2	Without varlny	Without varlnc	Without varlnw2	Without varlny	Without varlnc	Without varlnw2	Without varlny
eta0	6.724	0.416	-0.351	-16.871	2.865	-0.201	-28.401	2.674	-0.214	7.248	0.160	-0.338
eta1	0.690	0.095	-0.012	-6.349	1.091	-0.002	-12.781	1.106	-0.005	0.671	0.081	-0.012
eta2	-0.0002	0.00003	-0.00003	-0.00003	0.000001	-0.00002	0.0002	0.00002	-0.00002	-0.0002	0.00003	-0.00003
eta3	-0.003	-0.002	0.001	-0.002	-0.0002	0.001	-0.003	-0.00018	0.001	-0.003	-0.002	0.001
eta4	1.548	-0.018	0.029	0.376	0.060	0.006	0.280	0.061	0.012	1.534	-0.031	0.027
eta5	0.047	-0.004	-0.004	0.048	0.003	-0.001	0.087	0.0001	-0.003	0.047	-0.005	-0.004
eta6	0.008	0.000	0.000	0.004	0.001	0.000	0.010	0.001	0.0001	0.007	-0.00004	0.000
eta7	0.045	-0.029	0.174	0.034	-0.003	0.170	0.042	-0.004	0.165	0.053	-0.029	0.174
eta8	-0.005	0.018	-0.049	-0.006	0.001	-0.049	-0.005	0.001	-0.049	-0.006	0.018	-0.049
eta9	-0.815	-0.014	-0.005	-0.437	-0.018	-0.005	-0.693	-0.013	-0.003	-0.820	-0.007	-0.004
eta10	-0.052	-0.014	-0.001	-0.193	-0.007	-0.001	-0.362	-0.005	0.0002	-0.034	-0.016	-0.001
eta11	-0.010	0.006	-0.001	-0.005	0.001	-0.001	-0.010	0.001	-0.001	-0.010	0.006	-0.001
eta12	48.916	4.637	0.136	11.196	3.895	0.053	16.940	4.059	0.070	50.317	1.301	0.127
eta13	15.705	2.047	0.792	-6.037	1.028	0.530	-19.570	1.347	0.785	11.076	0.299	0.820
eta14	12.894	12.403	0.611	25.272	15.409	0.429	43.634	14.386	0.391	3.764	12.596	0.635
eta15	0.260	-0.998	0.008	-0.077	-1.070	0.001	0.488	-1.092	-0.001	0.477	-0.480	0.004
eta16	-15.187	0.433	0.045	-6.835	-0.145	-0.006	-12.799	-0.161	0.0002	-15.136	0.506	0.038
eta17	-12.145	-0.733	0.015	-3.109	3.083	0.043	-4.224	2.392	0.006	-12.236	-0.269	0.014
eta18	-8.486	-0.769	0.056	-8.584	-1.098	0.153	-16.450	-1.098	0.120	-10.527	-0.784	0.055
eta19	-0.344	-0.029	-0.007	-0.137	-0.018	-0.001	-0.258	-0.011	-0.001	-0.361	-0.017	-0.005
eta20	-565.460	25.800	-0.003	-197.627	7.249	0.001	-373.471	6.778	0.001	-556.863	25.620	-0.001
eta2125	-0.333	-0.005	-0.004	-0.400	-0.021	0.000	-0.647	-0.009	0.0003	-0.060	0.003	-0.003
eta 2224	0.048	0.028	0.001	-0.037	0.001	0.000	-0.124	0.001	-0.001	0.047	0.028	0.0004

Note: varlnc is the variance of quasi-fixed factor price; varlnw2 is the variance of variable input price; varlny is the variance of output level.

4.4.2 Impact of uncertainty on optimal factor allocations

4.4.2.1 Effect of quasi-fixed factor price uncertainty on investment demand

In the investment demand equation, the interaction term $\left[\left(\sigma_{\ln c}^{2}\right)_{t} \cdot K_{it-1}\right]$ is explored to examine the effect of quasi-fixed factor price uncertainty $\left(\sigma_{\ln c}^{2}\right)_{t}$ on investment demand, which differs across the range of the moderator variable, K_{it-1}. For investment demand, the conditional hypothesis of the two-way multiplicative interaction model is that investment is decreasing in quasi-fixed factor price uncertainty if and only if the lagged capital stock is positive. The estimate of the interaction term, θ_6 is significant; this suggests the presence of the interaction effect in the investment demand (see Table 4.19). Investment demand is negatively related to the variance of the quasi-fixed factor price in all scenarios, but this negative effect of quasi-fixed factor price uncertainty is increasing in scenario 1 compared to the benchmark and scenario 2. This result is in line with the theoretical findings of Dixit and Pindyck (1994), which have been empirically tested by Pietola and Myers (2000). The significant interaction term was further explored using the simple slope concept, i.e., by calculating the marginal effect of variance of quasi-fixed factor price on the investment demand, which is conditional on the values of the moderator variable (i.e., lagged quasi-fixed factor levels). Following Aiken and West (1991), the three values for the lag of quasi-fixed factor were selected to be one standard deviation above the mean (High lag *K*), at the mean, and one standard deviation below the mean (Low lag *K*), respectively.

For the benchmark case, Table 4.20 shows the marginal effect of the variance of quasi-fixed factor price on investment demand. The slope coefficient is negative and significant; this indicates a negative effect of uncertainty on investment demand. The negative marginal effect of quasi-fixed factor price uncertainty increases for the increase in the lagged quasi-fixed factor level. This result reveals a significant impact of uncertainty on the optimal factor allocation. In scenario 1 (change in τ_K), the slope coefficient is negative and significant, but the negative marginal effect of quasi-fixed factor price uncertainty is more pronounced for the higher values of the lag of quasi-fixed factor level compared to the benchmark scenario. Meanwhile, in scenario 2 (increase in $\sigma_{\ln c}^{2}$), the slope coefficient is negative and significant, but the negative effect of quasi-fixed factor price uncertainty is small compared to the benchmark scenario. This result indicates a negative relationship between the variance of quasi-fixed factor price $\left(\sigma^2_{\ln c}\right)$ and investment demand $\left(I\right)$ in all scenarios.

Table 4.19: Results of investment demand to examine interaction effect

	Benchmark			Scenario 2		Scenario 1		
	Known parameter values	Estimate	SE	Estimate	SE	Known parameter values	Estimate	SE
θ_0	2.5	2.5***	3.0E-06	2.5***	3.3E-06	2.5	2.5***	3.2E-06
θ_1	0.0005	0.0005***	3.5E-05	0.0005***	3.6E-05	0.001	0.001***	3.5E-05
θ_2	0.00175	0.00175***	9.6E-07	0.00175***	9.9E-07	0.0035	0.0035***	9.9E-07
θ_3	0.0005	0.0005***	2.8E-05	0.0005***	3.1E-05	0.001	0.001***	2.9E-05
θ_4	-0.00075	-0.00075***	2.1E-05	-0.00076***	2.3E-05	-0.0015	-0.0015***	2.2E-05
θ_5	0.08	0.08***	8.3E-09	0.08***	9.3E-09	0.11	0.11***	8.6E-09
θ_6	-0.5	-0.5***	8.7E-08	-0.5***	4.9E-08	-1	-1***	8.8E-08
θ_7	-1.5	-1.5***	4.2E-06	-1.5***	2.1E-06	-1.5	-1. 5***	4.2E-06

Note: Asterisks *** denote statistical significance at the 1% level; SE represents standard error.

Table 4.20: Impact of quasi-fixed factor price uncertainty on investment demand

	Levels of lag K	Slope parameters	SE
Benchmark	Low lag K (23.82)	-13.41***	2.67E-06
	Mean lag K (38.09)	-20.55***	2.28E-06
	High lag K (52.36)	-27.68***	2.53E-06
Scenario 1	Low lag K (22.43)	-23.93***	2.7E-06
	Mean lag K (35.93)	-37.43***	2.3E-06
	High lag K (49.43)	-50.93***	2.5E-06
Scenario 2	Low lag K (19.71)	-11.35***	1.4E-06
	Mean lag K (31.80)	-17.40***	1.2E-06
	High lag K (43.89)	-23.44***	1.2E-06

Note: Asterisks *** denote statistical significance at the 1% level; SE represents standard error.

4.4.2.2 Effect of variable input price uncertainty on variable input demand

The conditional hypotheses regarding the multiplicative interaction effect in the variable input demand are twofold. First, the impact of variance of the quasi-fixed factor price is negative on the variable input demand, given the combinations of investment and quasi-fixed factor levels. Second, the effect of variance of the variable input price is negative on the variable input demand, given the combinations of investment and level of quasi-fixed factors. For the variable input demand equation, the significant interaction terms in all scenarios indicate that the relation between input price uncertainty $\left(\sigma^2_{\ln c}\right.$ and $\left.\sigma^2_{\ln w_2}\right)$ and the variable input demand varies across levels of *K*, *I*, and/or combinations of *K* and *I*. This is achieved by using the simple slope concept, i.e., by finding the marginal effect of input price uncertainties on variable input demand. The marginal effect of variance of the quasi-fixed factor price $\left(\sigma^2_{\ln c}\right)$ on variable input demand $\left(x_2\right)$ is given by: $\partial x_2 / \partial \sigma^2_{\ln c} = \beta_{11} \cdot K_{it} + \beta_{14} \cdot I_{it} + \beta_{16}$. The marginal effect of the variance of variable input price $\left(\sigma^2_{\ln w_2}\right)$ on variable input demand $\left(x_2\right)$ is expressed as follows: $\partial x_2 / \partial \sigma^2_{\ln w_2} = \beta_{10} \cdot K_{it} + \beta_{15} \cdot I_{it} + \beta_{17}$, and the respective standard errors were calculated using equations (4.14) and (4.15) in section 4.2.2. The marginal effect (or simple slope coefficients) of input price uncertainties were further calculated for the combinations of investment $\left(I\right)$ and quasi-fixed factor $\left(K\right)$ levels (Table 4.21). The values for quasi-fixed factor and investment were selected to be one standard deviation above the mean (High *K* and High *I*), and one standard deviation below the mean (Low *K* and Low *I*), respectively.

In the benchmark model, the marginal effect of variance of the quasi-fixed factor price $\left(\sigma^2_{\ln c}\right)$ on variable input demand $\left(x_2\right)$ is positive for high values of *K* and *I,* but not significant (Table 4.22). This effect is positive and significant for the combination of low *K* and high *I* values. However, it is negative for the combinations of high *K* and low *I*, as well as for low *K* and low *I* values, but not significant. In scenario 1 (change in τ_K), the marginal effect of $\sigma^2_{\ln c}$ on x_2 is only negative for high *K* and low *I* values, but not significant. This effect is positive and significant for the rest of the combinations of *K* and *I*. In scenario 2 (change in $\sigma^2_{\ln c}$), the marginal effect of variance of the quasi-fixed factor price $\left(\sigma^2_{\ln c}\right)$ on the variable input demand $\left(x_2\right)$ is negative for all combinations of *K* and *I*. In scenario 3 (change in $\sigma^2_{\ln w_2}$), the results are the same as in the benchmark, revealing that the marginal effect of $\sigma^2_{\ln c}$ has no significant effect on x_2 when increasing the $\sigma^2_{\ln w_2}$ variable.

Table 4.21: Results of variable input demand to investigate interaction effects

Para-meters	Known parameter values	Benchmark			Scenario 2			Scenario 3			Scenario 1			
		Estimate		SE	Estimate		SE	Estimate		SE	Known parameters	Estimate		SE
β_0	2	2	***	0.112	2	***	0.071	2	***	0.049	2	2	***	0.032
β_1	-0.005	-0.0068	***	0.001	-0.007		0.007	-0.005	***	0.001	0.02	0.03	*	0.016
β_2	0.0024	0.0000		0.001	0.0013		0.001	0.0008		0.001	0.0033	0.0012		0.001
β_3	0.000015	-0.00278		0.002	-0.0015		0.004	-0.0008		0.001	0.000045	-0.019		0.065
β_4	0.0000525	-0.00002		0.000	-0.00006		0.000	-1E-05		0.000	0.000105	2E-06		0.000
β_5	0.000015	-0.01893		0.044	-0.02011		0.067	-0.0179		0.044	0.00003	-0.061		0.063
β_6	-0.0000225	0.00501		0.004	0.00628		0.006	0.0052		0.004	-4.5E-05	-0.021		0.022
β_7	-0.000125	-0.20473		0.185	-0.12085		0.131	-0.2049		0.185	-0.00025	-0.117		0.116
β_8	0.0000125	-0.00139		0.002	-0.00057		0.003	-0.0015		0.002	0.000025	0.032		0.024
β_9	0.00025	0.00058		0.001	0.00053		0.001	0.0006		0.001	0.0005	0.0011		0.001
β_{10}	-0.015	-0.015	***	0.000	-0.014	***	0.000	-0.015	***	0.000	-0.015	-0.01	**	0.003
β_{11}	-0.015	-0.006		0.004	-0.015	***	0.000	-0.0054		0.004	-0.03	-0.06	*	0.030
β_{12}	-0.018	0.002		0.009	-0.011	***	0.002	0.0012		0.009	-0.036	-0.023	***	0.005
β_{13}	-0.00025	0.0377		0.034	0.024		0.021	0.038		0.034	-0.0005	0.0037		0.020
β_{14}	0.3	0.3	***	0.004	0.3	***	0.003	0.3	***	0.004	0.6	0.6	***	0.036
β_{15}	-0.3	-0.3	***	0.000	-0.3	***	0.001	-0.3	***	0.000	-0.3	-0.3	***	0.004
β_{16}	-1.5	-1.2	***	0.179	-1.5	***	0.008	-1.2	***	0.179	-1.5	-1.2	***	0.181
β_{17}	-1	-1	***	0.002	-1	***	0.004	-1	***	0.001	-1	-1	***	0.003

Note: Asterisks ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively; SE represents standard error.

Table 4.22: Impact of quasi-fixed factor price uncertainty on variable input demand

Benchmark				**Scenario 1**			
		Slope parameters	SE			Slope parameters	SE
High K	High I			High K	High I		
15.16	5.07	0.18	2.2E-01	14.50	5.24	1.18 ***	3.6E-01
High K	Low I			High K	Low I		
15.16	3.28	-0.34	2.3E-01	14.50	2.38	-0.58	4.2E-01
Low K	High I			Low K	High I		
-15.09	5.07	0.35 ***	9.4E-02	-14.44	5.24	2.94 ***	6.0E-01
Low K	Low I			Low K	Low I		
-15.09	3.28	-0.17	1.0E-01	-14.44	2.38	1.18 *	5.1E-01
Scenario 2				**Scenario 3**			
		Slope parameters	SE			Slope parameters	SE
High K	High I			High K	High I		
12.30	3.99	-0.49 ***	1.5E-02	15.16	5.07	0.18	2.2E-01
High K	Low I			High K	Low I		
12.30	1.35	-1.26	1.2E-02	15.16	3.28	-0.34	2.3E-01
Low K	High I			Low K	High I		
-12.24	3.99	-0.13 ***	6.3E-03	-15.09	5.07	0.35 ***	9.4E-02
Low K	Low I			Low K	Low I		
-12.24	1.35	-0.90	4.2E-03	-15.09	3.28	-0.17	1.0E-01

Note: High K = mean +1SD; Low K = mean -1SD; and High I = mean +1SD;Low I = mean -1SD.
Asterisks *** and * denote statistical significance at the 1% and 10% levels, respectively;
SE represents standard error.

In the benchmark model, the marginal effect of the variance of variable input price $\left(\sigma^2_{\ln w_2}\right)$ on variable input demand $\left(x_2\right)$ is negative and significant for all combinations of K and I (Table 4.23). In scenario 1 (change in τ_K) and scenario 2 (change in $\sigma^2_{\ln c}$), the marginal effect of $\sigma^2_{\ln w_2}$ on x_2 is also negative and significant for all combinations of K and I. In scenario 3 (change in $\sigma^2_{\ln w_2}$), the results are the same as in the benchmark. This result reveals that the relationship between the variance of variable input price $\left(\sigma^2_{\ln w_2}\right)$ and the variable input demand $\left(x_2\right)$ is negative for all combinations of K and I values. The results in Table 4.23 indicate the relationship between the variance of the quasi-fixed factor price $\left(\sigma^2_{\ln c}\right)$ and the variable input demand $\left(x_2\right)$ is either positive or negative for different combinations of K and I values.

Table 4.23: Impact of variable input price uncertainty on variable input demand

Benchmark				Scenario 1			
		Slope parameters	SE			Slope parameters	SE
High *K*	High *I*			High *K*	High *I*		
15.16	5.07	-2.74 ***	0.003	14.50	5.24	-2.73 ***	0.032
High *K*	Low *I*			High *K*	Low *I*		
15.16	3.28	-2.21 ***	0.003	14.50	2.38	-1.86 ***	0.040
Low *K*	High *I*			Low *K*	High *I*		
-15.09	5.07	-2.29 ***	0.000	-14.44	5.24	-2.43 ***	0.062
Low *K*	Low *I*			Low *K*	Low *I*		
-15.09	3.28	-1.76 ***	0.000	-14.44	2.38	-1.57 ***	0.052
Scenario 2				**Scenario 3**			
		Slope parameters	SE			Slope parameters	SE
High *K*	High *I*			High *K*	High *I*		
12.30	3.99	-2.36 ***	0.008	15.16	5.07	-2.75 ***	0.002
High *K*	Low *I*			High *K*	Low *I*		
12.30	1.35	-1.57 ***	0.008	15.16	3.28	-2.21 ***	0.002
Low *K*	High *I*			Low *K*	High *I*		
-12.24	3.99	-2.01 ***	0.002	-15.09	5.07	-2.29 ***	0.000
Low *K*	Low *I*			Low *K*	Low *I*		
-12.24	1.35	-1.22 ***	0.001	-15.09	3.28	-1.76 ***	0.000

Note: High *K* = mean +1SD; Low *K* = mean -1SD; and High *I* = mean +1SD; Low *I* = mean -1SD.
Asterisks *** denote statistical significance at the 1% level; SE represents standard error.

Similar to the two-way interaction effect, the significance of the three-way interaction term indicates that the relationship between uncertainty and factor demands (investment and variable input demand equations) varies across levels of the two moderator variables and/or combinations of moderator variables. Further, this significant three-way interaction is the result of significant differences between any two, three or all four combinations of the two moderator variables at both high and low levels. Statistical probing is required to find out whether any difference between pairs of slopes is significant or whether an individual slope is a significant predictor of the dependent variable. Three-way multiplicative interaction effects are not significant for the numeraire demand function; hence, these results have not been presented here.

5 Conclusions

Existing dynamic dual models of efficiency typically assume static price expectations and ignore production uncertainty when deriving dynamic efficiency measures, whereas dynamic dual models of investment consider non-static price expectations, but disregard inefficiency measures when deriving optimal decision rules. In this thesis, a model of dynamic efficiency under uncertainty has been developed by merging the stochastic model of investment under uncertainty with (deterministic) dynamic efficiency analysis. The derived model extends existing dynamic efficiency approaches, since it accounts for non-static expectations of factor prices and output. A stochastic dynamic programming technique has been used to characterize duality relations between variable cost and optimal value functions.

For this purpose, two types of cost functions have been created and expressed in terms of behavioral and actual value functions. Using shadow input prices and quantities sets up a behavioral value function guaranteeing a cost-minimized relation under shadow prices. Further, observed input prices and quantities were used to set up the actual value function. By assuming that actual input quantities are at the optimal levels, the actual value function becomes the optimized actual value function and represents fully efficient input use. That is, in the presence of perfect efficiency, the optimized actual value function is equivalent to the behavioral value function, whereas in the presence of inefficiency they differ. As a result, the optimized actual value function is expressed in terms of the behavioral value function, which allows one to express the optimized actual factor demand equations in terms of the behavioral value function, including several inefficiency terms. The resulting theoretical factor demand equations could then serve as a starting point for the empirical model derivation undertaken here. Therefore, a functional form for the behavioral value function is specified, which fulfils the properties specified by Pietola and Myers (2000), to further account for output and input price uncertainty in the optimal factor demand equations. The resulting stochastic factor demand equations were then employed in the econometric estimation of technical and allocative efficiency.

The theoretical model derivation findings have been illustrated using simulations. In simulations, to reduce a complex theoretical model structure, the specified value function is reduced to one quasi-fixed factor, one output, and two variable inputs. Results might vary depending upon the number of quasi-fixed factors that are considered. The resulting model was applied to a sample of simulated data, and 1000 replications were used to obtain estimates of the simulated factor demand equations.

To check for the presence of omitted variables, a RESET test was used, but this test result was misleading for the numeraire input demand equation, again proving that the test itself is very weak, as often mentioned in the literature (Leung and Yu 2000). The simulation results imply that disregarding uncertainty in factor demand equations results in biased estimates. A change in the TI parameter of net investment (scenario1) results in increased bias on the estimated coefficients in factor allocations. The simulation results also show that there is a negative effect of quasi-fixed factor price uncertainty on investment demand for different values of the quasi-fixed factor level. On the contrary, variance of the variable input price has a negative marginal effect on variable input demand for combinations of values for investment and quasi-fixed factor levels.

In this thesis, the static shadow cost approach has been developed in the context of a stochastic dual model of investment under uncertainty. This theoretical approach entails some caveats. First, the derived theoretical model in this thesis did not explicitly account for technical change. Hence, further research should accommodate technical change while deriving new theoretical models. Considering technical change when deriving new models can facilitate the development of a dynamic productivity model in the presence of uncertainty. Second, adjustment costs have been expressed by simple adjustment rates that may be transformed into a flexible linear accelerator model, though it has been emphasized in the investment literature that more sophisticated adjustment cost functions are required to appropriately specify investment demand functions (cf. Hüttel et al. 2010). In addition, this study has proposed a dynamic efficiency model under uncertainty using a dual cost approach; this can be extended to develop a dynamic efficiency model under uncertainty using a dual profit approach if prices and quantities of inputs and outputs are available. Future research should address these lacunae.

The simulation results of this thesis are also subject to limitations. First, the identification problem has been more severe in the variable input demand equations compared to the investment demand equation. Second, the simulations do not account for the influence of co-variance variables on investment and variable input demands. Third, to quantify bias on the estimates of the model parameters, factor demands have been estimated using reduced-form equations in the simulations. Consequently, the quantified bias may differ when using a non-linear system of factor demand equations. Finally, the simulations do not consider firm- and time-specific inefficiency measures. The omitted variable bias results for the simulations may therefore change if time- and firm-specific inefficiency measures are considered in the estimation of factor demand equations.

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Appendices

Appendix A: Uncertainty term

The behavioral uncertainty term is given as

$$\Omega^b = \sum_{j=1}^{1+\bar{n}+\bar{m}} \sum_{j'=1}^{1+\bar{n}+\bar{m}} J^b_{z_j z_{j'}} \sigma_{jj'}\,, \tag{A.1}$$

where $J^b_{z_j z_{j'}}$ denotes second partial derivatives of J^b with respect to z. The term $\sigma_{jj'}$ represents the variance and co-variance parameters. Indices j and j' represents the respective state variables in z. The state vector $z(t)$, consists of the logarithms of the output level and input prices, i.e., $z(t) = \left[\ln y(t), \ln w_n(t), \ln c_m(t)\right]$. Using the specified functional form for the behavioral value function [in equation (3.25)], the uncertainty term is derived as:

$$\begin{aligned}
\Omega^b = {} & A_{yy}\sigma^2_{\ln y} + 2\sum_n A_{w_n y}\sigma_{\ln w_n, \ln y} + 2\sum_m A_{c_m y}\sigma_{\ln c_m, \ln y} \\
& + \sum_n \left(A_{w_n w_n}\sigma^2_{\ln w_n} + \sum_m \left(A_{w_n K_m} K_m \right) \cdot w^b_n \sigma^2_{\ln w_n} \right) + \sum_n \left(\sum_m A_{c_m w_n} \sigma_{\ln c_m, \ln w_n} \right) \\
& + \sum_m \left(\sum_n A_{c_m w_n} \sigma_{\ln c_m, \ln w_n} \right) + \sum_m \left(A_{c_m c_m}\sigma^2_{\ln c_m} + \sum_m \left(M^{-1}_{c_m K_m} K_m \right) \cdot c_m \sigma^2_{\ln c_m} \right)
\end{aligned} \tag{A.2}$$

where A-parameters are the second order parameters of the value function, and the variable σ denote respective variance and co-variance of the stochastic variables.

Appendix B: Model specification for the three-way interaction effect

The numeraire input demand equation is given as:

$$\begin{aligned}(x_1)_{it} &= \eta_0 + \eta_1 \cdot K_{it} + \eta_2 \cdot (K^2)_{it} + \eta_3 \cdot \ln y_{it} \cdot K_{it} + \eta_4 \cdot (w_{21})_t \cdot K_{it} + \eta_5 \cdot c_t \cdot K_{it} + \eta_6 \cdot K_{it} \cdot I_{it} \\ &+ \eta_7 \cdot \ln y_{it} + \eta_8 \cdot (\ln y_{it})^2 + \eta_9 \cdot \ln y_{it} \cdot \ln(w_{21})_t + \eta_{10} \cdot \ln y_{it} \cdot \ln c_t + \eta_{11} \cdot \ln y_{it} \cdot I_{it} \\ &+ \eta_{12} \cdot \ln(w_{21})_t + \eta_{13} \cdot (\ln(w_{21})_t)^2 + \eta_{14} \cdot \ln(w_{21})_t \cdot \ln c_t + \eta_{15} \cdot (w_{21})_t \cdot (x_2)_{it} \\ &+ \eta_{16} \cdot (w_{21})_t \cdot I_{it} + \eta_{17} \cdot \ln c_t + \eta_{18} \cdot (\ln c_t)^2 + \eta_{19} \cdot c_t \cdot I_{it} + \eta_{20} \cdot I_{it} + \eta_{21} \cdot K_{it} \cdot c_t \cdot (\sigma^2_{\ln c})_t \\ &+ \eta_{22} \cdot K_{it} \cdot (w_{21})_t \cdot (\sigma^2_{\ln w_2})_t + \eta_{23} \cdot (\sigma^2_{\ln y})_t + \eta_{24} \cdot (\sigma^2_{\ln w_2})_t + \eta_{25} \cdot (\sigma^2_{\ln c})_t \\ &+ \eta_{27} \cdot K_{it} \cdot (\sigma^2_{\ln c})_t + \eta_{28} \cdot c_t \cdot (\sigma^2_{\ln c})_t + \eta_{29} \cdot K_{it} \cdot (\sigma^2_{\ln w_2})_t + \eta_{30} \cdot (w_{21})_t \cdot (\sigma^2_{\ln w_2})_t \end{aligned} \tag{B.1}$$

To explore the effect of input price and output uncertainty on the numeraire input demand equation requires model specification for the three-way interaction effects. Following Dawson and Richter (2006) procedure to test the joint effect of three independent variables on a dependent variable. This subsection provides the model specification for finding the impact of input prices uncertainty on the numeraire demand equation.

First, differentiating the numeraire input demand equation (B.1) with respect to the variance of quasi-fixed factor price, results in the slope (marginal effect) of $\sigma^2_{\ln c}$ on numeraire input demand, x_1:

$$\frac{\partial x_1}{\partial \sigma^2_{\ln c}} = \eta_{21} \cdot K_{it} \cdot c_t + \eta_{25} + \eta_{27} \cdot K_{it} + \eta_{28} \cdot c_t \tag{B.2}$$

The standard error for the slope of $\sigma^2_{\ln c}$ on x_1 is given by:

$$\mathrm{SE}(x_1)_{\sigma^2_{\ln c}} = \sqrt{\begin{aligned}&(K_{it})^2 (c_t)^2 \cdot \mathrm{var}(\hat{\gamma}_{21}) + \mathrm{var}(\hat{\gamma}_{25}) + (K_{it})^2 \cdot \mathrm{var}(\hat{\gamma}_{27}) + (c_t)^2 \cdot \mathrm{var}(\hat{\gamma}_{28}) \\ &+2K_{it} \cdot \mathrm{cov}(\hat{\gamma}_{21}\hat{\gamma}_{25}) + 2c_t \cdot \mathrm{cov}(\hat{\gamma}_{27}\hat{\gamma}_{25}) + 2K_{it}c_t \cdot \mathrm{cov}(\hat{\gamma}_{28}\hat{\gamma}_{25}) \\ &+2K_{it}c_t \cdot \mathrm{cov}(\hat{\gamma}_{27}\hat{\gamma}_{28}) + 2c_t (K_{it})^2 \cdot \mathrm{cov}(\hat{\gamma}_{21}\hat{\gamma}_{27}) + 2K_{it} (c_t)^2 \cdot \mathrm{cov}(\hat{\gamma}_{21}\hat{\gamma}_{28})\end{aligned}} \tag{B.3}$$

The slope of $\sigma^2_{\ln c}$ on x_1 for the different combinations of *K* and *c*—are formulated here. Therefore, the slopes of the four lines can be represented by

$$\eta_{21} \cdot K_{High} \cdot c_{High} + \eta_{25} + \eta_{27} \cdot K_{High} + \eta_{28} \cdot c_{High} \qquad \cdots(1)$$

$$\eta_{21} \cdot K_{High} \cdot c_{Low} + \eta_{25} + \eta_{27} \cdot K_{High} + \eta_{28} \cdot c_{Low} \qquad \cdots(2)$$

$$\eta_{21} \cdot K_{Low} \cdot c_{High} + \eta_{25} + \eta_{27} \cdot K_{Low} + \eta_{28} \cdot c_{High} \qquad \cdots(3)$$

$$\eta_{21} \cdot K_{Low} \cdot c_{Low} + \eta_{25} + \eta_{27} \cdot K_{Low} + \eta_{28} \cdot c_{Low} \qquad \cdots(4)$$

Here, the difference between slopes is calculated by using the slopes of the lines given in (1)–(4). The formulas for the differences among all six pairs of slopes, is as follows:

$$\begin{array}{ll}
(1)-(2) & \eta_{21} \cdot K_{High} \cdot \left(c_{High} - c_{Low}\right) + \eta_{28} \cdot \left(c_{High} - c_{Low}\right) \\
(1)-(3) & \eta_{21} \cdot \left(K_{High} - K_{Low}\right) \cdot c_{High} + \eta_{27} \cdot \left(K_{High} - K_{Low}\right) \\
(2)-(4) & \eta_{21} \cdot \left(K_{High} - K_{Low}\right) \cdot c_{Low} + \eta_{27} \cdot \left(K_{High} - K_{Low}\right) \\
(3)-(4) & \eta_{21} \cdot K_{Low} \cdot \left(c_{High} - c_{Low}\right) + \eta_{28} \cdot \left(c_{High} - c_{Low}\right) \\
(1)-(4) & \eta_{21} \cdot \left(K_{High} \cdot c_{High} - K_{Low} \cdot c_{Low}\right) + \eta_{27} \cdot \left(K_{High} - K_{Low}\right) + \eta_{28} \cdot \left(c_{High} - c_{Low}\right) \\
(2)-(3) & \eta_{21} \cdot \left(K_{High} \cdot c_{Low} - K_{Low} \cdot c_{High}\right) + \eta_{27} \cdot \left(K_{High} - K_{Low}\right) + \eta_{28} \cdot \left(c_{Low} - c_{High}\right)
\end{array}$$

The standard errors of slope differences are as follows:

$$\begin{array}{ll}
(1)-(2) & 2\sqrt{\operatorname{var}\left(\hat{\gamma}_{21}\right) + \operatorname{var}\left(\hat{\gamma}_{28}\right) + 2\operatorname{cov}\left(\gamma_{21}\gamma_{28}\right)} \\
(1)-(3) & 2\sqrt{\operatorname{var}\left(\hat{\gamma}_{21}\right) + \operatorname{var}\left(\hat{\gamma}_{27}\right) + 2\operatorname{cov}\left(\gamma_{21}\gamma_{27}\right)} \\
(2)-(4) & 2\sqrt{\operatorname{var}\left(\hat{\gamma}_{21}\right) + \operatorname{var}\left(\hat{\gamma}_{27}\right) - 2\operatorname{cov}\left(\gamma_{21}\gamma_{27}\right)} \\
(3)-(4) & 2\sqrt{\operatorname{var}\left(\hat{\gamma}_{21}\right) + \operatorname{var}\left(\hat{\gamma}_{28}\right) - 2\operatorname{cov}\left(\gamma_{21}\gamma_{28}\right)} \\
(1)-(4) & \gamma_{21} \cdot \left(K_{High} \cdot c_{High} - K_{Low} \cdot c_{Low}\right) + \gamma_{27} \cdot \left(K_{High} - K_{Low}\right) + \gamma_{28} \cdot \left(c_{High} - c_{Low}\right) \\
(2)-(3) & \gamma_{21} \cdot \left(K_{High} \cdot c_{Low} - K_{Low} \cdot c_{High}\right) + \gamma_{27} \cdot \left(K_{High} - K_{Low}\right) + \gamma_{28} \cdot \left(c_{Low} - c_{High}\right)
\end{array}$$

Second, differentiating the numeraire input demand equation (B.1) with respect to the variance of variable input price, results in the slope of $\sigma^2_{\ln w_2}$ on numeraire input demand, x_1:

$$\frac{\partial x_1}{\partial \sigma^2_{\ln w_2}} = \eta_{22} \cdot K_{it} \cdot \left(w_{21}\right)_t + \eta_{24} + \eta_{29} \cdot K_{it} + \eta_{30} \cdot \left(w_{21}\right)_t \qquad \text{(B.4)}$$

The standard error formula for the slope of $\sigma^2_{\ln w_2}$ on x_1, is as follows:

$$\mathrm{SE}(x_1)_{\sigma^2_{\ln w_2}} = \sqrt{\begin{aligned}&(K_{it})^2 (w_{21})_t^2 \cdot \mathrm{var}(\hat{\gamma}_{22}) + \mathrm{var}(\hat{\gamma}_{24}) + (K_{it})^2 \cdot \mathrm{var}(\hat{\gamma}_{29}) \\ &+(w_{21})_t^2 \cdot \mathrm{var}(\hat{\gamma}_{30}) + 2K_{it} \cdot \mathrm{cov}(\hat{\gamma}_{29}\hat{\gamma}_{24}) + 2(w_{21}) \cdot \mathrm{cov}(\hat{\gamma}_{30}\hat{\gamma}_{24}) \\ &+2K_{it}(w_{21}) \cdot \mathrm{cov}(\hat{\gamma}_{22}\hat{\gamma}_{24}) + 2K_{it}(w_{21})_t \cdot \mathrm{cov}(\hat{\gamma}_{29}\hat{\gamma}_{30}) \\ &+2(w_{21})_t (K_{it})^2 \cdot \mathrm{cov}(\hat{\gamma}_{22}\hat{\gamma}_{29}) + 2K_{it}(w_{21})_t^2 \cdot \mathrm{cov}(\hat{\gamma}_{22}\hat{\gamma}_{30})\end{aligned}} \tag{B.5}$$

The slope of $\sigma^2_{\ln w_2}$ on x_1 for the different combinations of K and w_{21}—are formulated here. Therefore, the slopes of the four lines can be represented by

$$\eta_{22} \cdot K_{High} \cdot (w_{21})_{High} + \eta_{24} + \eta_{29} \cdot K_{High} + \eta_{30} \cdot (w_{21})_{High} \qquad \cdots(5)$$

$$\eta_{22} \cdot K_{High} \cdot (w_{21})_{Low} + \eta_{24} + \eta_{29} \cdot K_{High} + \eta_{30} \cdot (w_{21})_{Low} \qquad \cdots(6)$$

$$\eta_{22} \cdot K_{Low} \cdot (w_{21})_{High} + \eta_{24} + \eta_{29} \cdot K_{Low} + \eta_{30} \cdot (w_{21})_{High} \qquad \cdots(7)$$

$$\eta_{22} \cdot K_{Low} \cdot (w_{21})_{Low} + \eta_{24} + \eta_{29} \cdot K_{Low} + \eta_{30} \cdot (w_{21})_{Low} \qquad \cdots(8)$$

The slope differences are calculated by using the slopes of the lines given in (5)–(8). The formulas for the differences among all six pairs of slopes, is as follows:

$$\begin{aligned}
&(5)-(6) \qquad \eta_{22} \cdot K_{High} \cdot \left[(w_{21})_{High} - (w_{21})_{Low}\right] + \eta_{30} \cdot \left[(w_{21})_{High} - (w_{21})_{Low}\right] \\
&(5)-(7) \qquad \eta_{22} \cdot (K_{High} - K_{Low}) \cdot (w_{21})_{High} + \eta_{29} \cdot (K_{High} - K_{Low}) \\
&(6)-(8) \qquad \eta_{22} \cdot (K_{High} - K_{Low}) \cdot (w_{21})_{Low} + \eta_{29} \cdot (K_{High} - K_{Low}) \\
&(7)-(8) \qquad \eta_{22} \cdot K_{Low} \cdot \left[(w_{21})_{High} - (w_{21})_{Low}\right] + \eta_{30} \cdot \left[(w_{21})_{High} - (w_{21})_{Low}\right] \\
&(5)-(8) \qquad \eta_{22} \cdot \left[K_{High} \cdot (w_{21})_{High} - K_{Low} \cdot (w_{21})_{Low}\right] + \eta_{29} \cdot (K_{High} - K_{Low}) + \eta_{30} \cdot \left[(w_{21})_{High} - (w_{21})_{Low}\right] \\
&(6)-(7) \qquad \eta_{22} \cdot \left[K_{High} \cdot (w_{21})_{Low} - K_{Low} \cdot (w_{21})_{High}\right] + \eta_{29} \cdot (K_{High} - K_{Low}) + \eta_{30} \cdot \left[(w_{21})_{Low} - (w_{21})_{High}\right]
\end{aligned}$$

In the numeraire input demand equation (B.1), the three-way interaction effects are not significant; therefore, the significance tests of differences between pairs of slopes are not calculated here.

Appendix C: Flexible accelerator model

The multivariate flexible accelerator form is as follows:

$$\dot{K}_m^b = N\left(K_m - \bar{K}_m\right), \tag{C.1}$$

where $\dot{K}_m^b$ is the behavioral net investment demand, N denotes the adjustment matrix of the accelerator mechanism, K_m represents current level of quasi-fixed factor and $\bar{K}_m$ denotes long-run steady state quasi-fixed factor stock.

Since one quasi-fixed factor is considered in simulation, therefore, substituting $m = 1$ in equation (C.1) results in

$$\dot{K}_1^b = N\left(K_1 - \bar{K}_1\right), \tag{C.2}$$

where N denotes the partial adjustment coefficient that signifies how quickly the current level of quasi-fixed factor stock, K_m, will adjust to the long-run steady state quasi-fixed factor stock, $\bar{K}_m$.

Now, the behavioral net investment demand equation (3.26) for the one quasi-fixed factor case is rewritten as:

$$\dot{K}^b = \left(M_{cK}^{-1} \cdot c\right)^{-1} r\left(b_c + A_{cy} \cdot \ln y + A_{cw_2} \cdot \ln w_2^b + A_{cc} \cdot \ln c\right) + \left(r - M_{cK} - \frac{1}{2} \cdot \sigma_{\ln c}^2\right) \cdot K \tag{C.3}$$

Substituting the behavioral net investment demand equation in the equation (C.2) yields long-run steady state demand function for the quasi-fixed factor:

$$\bar{K} = -\left(M_{cK} \cdot c^{-1}\right) r \left(r - M_{cK} - \frac{1}{2} \cdot \sigma_{\ln c}^2\right)^{-1} \left(b_c + A_{cy} \cdot \ln y + A_{cw_2} \cdot \ln w_2^b + A_{cc} \cdot \ln c\right), \tag{C.4}$$

where $\left(r - M_{cK} - \frac{1}{2} \cdot \sigma_{\ln c}^2\right)$ term denotes adjustment rate, i.e., $\left(r - M_{cK} - \frac{1}{2} \cdot \sigma_{\ln c}^2\right) = N$. This term is used to calculate the adjustment rate in simulation.

Table C.1: Interpretation of adjustment rates in the previous dynamic dual models

Author (year)	Adjustment rate formula a)	Application	Discount rate	Quasi-fixed factors	Adjustment rate	Interpretation
Cost-minimization problems						
Epstein and Denny (1983)	Flexible accelerator form: $\dot{K}^{*} = M\left[K - \bar{K}\right]$, where $M = r - A_{pK}$ is the adjustment matrix $\bar{K}$ = steady state capital stock K = quasi-fixed factor $\dot{K}^{*}$ = optimal net investment demand	U.S. manu-facturing	0.07	Capital Labor	12% per year 90% per year	For capital, only 12% of the adjustment occurs in one year. If capital is at its steady state level, then 90% of the adjustment to any desired change in the stock of labor occurs within one year.
Stefanou et al. (1992)	Multivariate flexible accelerator form: $\dot{K}^{*} = \left(ru - M\right)\left(K - \bar{k}\right)$ $\left(ru - M\right)$ = adjustment matrix $\bar{k}$ = steady state stock of quasi-fixed factors r = discount rate u = identity matrix	German dairy	0.1	Capital, Land, Family labor	-0.545 -0.414 -0.383	To adjust to long-run equilibrium levels, capital, land, and family labor may take approximately 2, 2.5 and 3 years, respectively.
Pietola and Myers (2000)	Flexible accelerator model: $\dot{K} = N\left(K - \bar{K}\right)$ $\bar{K}$ = steady state capital stock N = matrix of adjustment rates determined by the estimated value of M and γ_{labor}.	Finnish pork industry	0.05	Labor, Machinery, Real estate		$\hat{N} = \begin{bmatrix} -0.0203 & & \\ 0.0444 & -0.0291 & \\ 0.0035 & 0.00887 & -0.0749 \end{bmatrix}$ In a labor contraction phase, the difference between current labor use and steady state use would decline by only 32% over a 5 year period.

Table C.1: Interpretation of adjustment rates in the previous dynamic dual models (cont.…)

Author (year)	Adjustment rate formula [a)]	Application	Discount rate	Quasi-fixed factors	Adjustment rate	Interpretation
Cost minimization problems						
Rungsuriyawiboon and Stefanou (2007)	$\left[r-\left(A^{ck}\right)^{-1}\right]$ r = discount rate A^{ck} = second order parameter of value function	U.S. electric utilities	0.05	Capital	3%	The capital stock adjusts 3% per annum to the long-run equilibrium levels. This sluggish adjustment is due to the non-storable characteristic of electricity and capital specific nature of utility investments.
Serra et al. (2010)	$\left(r\mathbf{U}-\mathbf{M}\right)$ is the adjustment matrix r = discount rate $\mathbf{U}$ = identity matrix $\mathbf{M}$ = second-order matrix of quasi-fixed factors	U.S. agriculture	0.055	Labor Aggregate measure of capital	-0.114 -0.022	Labor requires about 9 years adjusting to long-run equilibrium. Composite capital index (includes land and machinery) requires around 46 years to long-run equilibrium. This slow adjustment of capital may be partly due to land and credit market rigidities.
Rungsuriyawiboon and Hockmann (2012)	$M_u=\left[r-\left(\beta_{qp_q}\right)^{-1}\right]$ r = discount rate A^{ck} = second order parameter of value function q = quasi-fixed factors (capital and labor) p = price of quasi-fixed factors	Polish agriculture	0.05	Capital, Land	4% per annum 3.6% per annum	In the Northwest farms, capital approximately takes 25 years and land takes 28 years to adjust fully to its long-run equilibrium level. This results imply that the sluggish adjustment process exist in Polish agriculture.

Table C.1: Interpretation of adjustment rates in the previous dynamic dual models (cont.…)

Author (year)	Adjustment rate formula [a)]	Application	Discount rate	Quasi-fixed factors	Adjustment rate	Interpretation
Profit maximization problems						
Vasavada and Chambers (1986)	Adjustment matrix $M = (ru + D)$ r = discount rate u = identity matrix D = second-order matrix of quasi-fixed factors	U.S. agriculture		Labor Capital Land	-0.069 -0.118 -0.587	These values were consistent with about 10 year adjustment lags for capital stocks, while land stocks adjusted to desired levels in less than 2 years. The eigenvalues of the accepted model are 0.954, 0.832, and 0.437 and are consistent with stable adjustment.
Howard and Shumway (1988)	Adjustment rate for cows = $(M_{11} = B_{11} + r)$ Adjustment rate for labor = $(M_{22} = B_{22} + r)$	U.S. dairy industry	0.03	Cow Labor	-0.09 -0.397	Both cows and labor exhibited quasi-fixity. That is, the estimated adjustment rate implies that cow numbers adjust 9% of the way towards long-run optimal levels in one year and labor adjusts 40%.

a) For further details on the adjustment rate formulas please refer to the original articles.